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ABSTRACT

The standards developed for reform in mathematics education by the National Council of Teachers of Mathematics (NCTM) de-emphasize the algorithmic manipulation of mathematical symbols. The NCTM takes the position that the rapidly changing workplace is an environment that requires problem-solving skills with computers being used to perform routine mathematical computations. This report describes the computer-based information curriculum module developed for beginning algebra to meet the challenges of the proposed reform. The RAND Algebra Tutor Project has been developing and piloting intelligent computer tools for first-year algebra students. Varios problems led to the development of a full curriculum in order to successfully implement computer-assisted instruction. The curriculum departed from traditional algebra instructions in these ways: the pedagogy stressed situational learning in real-life problems, introducing a conceptual framework before local skills practice and used indictive instruction and cooperative learning. The curriculum achieved mixed effectiveness. In general, students were more successful at creating and using qualitative models than quantitative models. Appendix A contains the 12 teacher lessons plans for the linear equations unit and related student materials. Appendix B contains the evaluation measures for the arithmetic text, pre/post tests, student course evaluations, and unit evaluations. (Contains 36 references.) (MDH)

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A RAND NOTE

Implementing a Novel Computer-Related Algebra Course

Abby Robyn, Cathleen Stasz,
David McArthur, Tor Ormseth,
Matthew Lewis

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Implementing a Novel Computer-Related Algebra Course

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Supported by the
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RAND

PREFACE

The research reported in this Note is part of an ongoing series of studies to develop and field test an intelligent computer tutor for basic algebra and describes, in particular, a model for transferring technology into the classroom. This project was supported by RAND, using its own funds; and it incorporated software developed with a grant from the National Science Foundation as a part of its Applications of Advanced Technologies Program.

SUMMARY

The crisis in education articulated in *A Nation At Risk* (1983) and the Carnegie report (1986) has focused extensively on the poor mathematics performance of American students. This concern is reflected in the 1986 National Assessment of Education Progress (Dossey et al., 1988) and in the complaints of employers that workers do not have appropriate skills to contribute successfully to a technology-dominated workplace (Romberg, 1987). In response to the call for reform, the National Council of Teachers of Mathematics (NCTM) proposed a new set of standards for K-12 mathematics instruction (Romberg, 1987). It suggests a variety of fundamental content changes for mathematics education. At the most general level, the NCTM standards emphasize the need for students to learn *higher-order thinking* and *problem-solving skills*. Conversely, the standards deemphasize the rote or algorithmic manipulation of mathematical symbols, arguing both that the rapidly changing workplace demands the acquisition of flexible problem-solving skills and that computers should take over the computation of routine mathematical results.

The NCTM standards lead to several specific questions concerning the content of mathematics courses and the role of computers in the classroom. What problem-solving and higher-order thinking skills should replace traditional ones in the classroom? What techniques should be used to teach these new skills? How should computers be introduced into these courses to support students' thinking by doing routine mathematical computations? Should computers be used simply as a problem-solving resource or should they take a more active role in teaching students?

The RAND Algebra Tutor Project has been developing and piloting intelligent computer tools for first-year algebra students. However, inadequate training, new instructional, pedagogical, and organizational goals, and logistical problems all contribute to the difficulty of effectively implementing computer-assisted instruction. It has become clear to us that successful implementation of a technology-based course requires the development of a full curriculum, comprising a set of support materials that permit the classroom culture to change to take advantage of the tools and new curriculum content. In this Note, we describe the curriculum module for beginning algebra that we developed to meet the challenges discussed above.

THE CURRICULUM

The curriculum we developed departed from traditional algebra instruction in three ways: The pedagogy stressed new techniques situating learning in real-life problems, introduced a conceptual framework before local skills practice, and used inductive instruction and cooperative learning; the course content included innovative topics-modeling, covariate relationships, and problem-solving strategies; and the teaching tools included daily access to an intelligent tutoring system. A multidisciplinary team composed of computer scientists, cognitive and educational psychologists, implementation and curriculum experts, and teachers developed the cognitive goals for the course and the course materials. Curriculum materials included teacher lesson plans, parallel computer and non-computer activities for students, and evaluation instruments.

Content Goals

The course subject matter centered on teaching students various *model-building* skills, similar to those noted in NCTM and other related research (e.g., Fey, 1984; Thompson and Rathmell, 1988; Usiskin, 1985). By model-building skills we mean the ability to see how formal mathematical objects can relate to the real world—for example, how equations can provide powerful descriptions, or models, of the interactions among real-world objects or properties. Model building includes several skills rarely taught in traditional classrooms: formulating which objects or properties of a real-world situation should be represented as mathematical objects or variables; data gathering, representation, and building qualitative models; translating qualitative models into quantitative ones; and making inferences about the behavior of variables.

Pedagogical Goals

The pedagogical goals adhered to the principles that problem solving should reflect the real world and take advantage of computer capabilities. Thus, coursework problems should be grounded in natural situations such as after school jobs, rock concerts, and pollution, and problem solving should reflect the cooperative arrangements frequently found in the workplace. Moreover, whenever possible, learning should occur through induction rather than didactic presentation, and global skills such as strategic goals for problem solving should be taught before the skills of symbol manipulation.

Computer Tools

Two computer environments were built to help students engage in each of the main activities involved in model building, formulation, data gathering and representation, translation, and inferencing using equations. Curriculum software was implemented in Franz Lisp on Sun Microsystems workstations (see McArthur et al. 1987; McArthur and Stasz, 1990, for a more complete description of the tutor architecture and functionality). For each assignment, students select a situation from a menu of options (e.g., video store job, pizza party, weight estimation, boating). Students determine the variables in a situation and insert them into a word graph which forms a qualitative model of the variable relationships. Besides the initial situation and word graph, the model-building computer environment provides several tools to help the student gather and organize data, including tables of values and cartesian graphs. Problems posed by the computer are aimed at helping students develop an intuitive or qualitative model of situations which in later sessions will be translated into a more quantitative, equational representation.

In later computer sessions, additional tools appear to help students construct operators and then symbolic equations. The students then use their equational models to generate answers to problems about the topic they have chosen. It is at this point that an equation-solving intelligent tutor system (ITS) comes into play, coaching the student through the manipulations required to solve equations and make the desired inference. Specific assignments are structured to focus on what goal to pursue in solving an equation (e.g., isolating a variable from its coefficient), what operation implements the goal (e.g., dividing both sides of the equation by 5), and how to manipulate symbols to accomplish an operation. Consistent with our top-down pedagogical philosophy, in equation-solving assignments, students first focus only on selecting appropriate goals, and the ITS supplies operations and manipulations. Later, students become responsible for operations, and in the final assignments students do goals and manipulations.

Teacher and Student Materials

To provide support for teachers and insure that the course material would be taught as intended, we developed specific lesson plans (see Appendix A for the full set of curriculum materials). Each lesson plan contains the goals for the lesson and an almost scripted set of activities to carry out, including board examples for the teacher to use. In addition, eight computer and 12 noncomputer assignment worksheets were developed.

IMPLEMENTATION

The curriculum module was taught by two teachers who were consultants to the curriculum development team. Participating students were enrolled in either Math A or regular algebra classes. Math A is the first year of a new two-year course implementing the recent California Math Framework (California State Department of Education, 1985). It features several discrete units—probability, symmetry, statistics, algebra, and geometry. The Math A course is intended to expose low-achieving mathematics students to enough algebra and geometry to enable them to continue in a college-focused academic plan if they so choose. To provide comparative information, four regular algebra classes were asked to participate in the data-collection activities.

Training

Since both pilot teachers were members of the curriculum development team, formal teacher training was required only for the computer component of the course. The software was available for teacher review as it was developed. Additionally, some assignments were piloted with students in a lab situation. Seven Sun 3/50 workstations were installed in the Math A classroom. Project staff were present in every class to handle any computer operational problems that arose.

Assessment

We used multiple methods to assess the curriculum, including mathematics and algebra achievement tests, two unit tests, background and attitude questionnaires, classroom observation notes, and computer records (see Appendix B for the assessment instruments).

EVALUATION AND CONCLUSIONS

This study had two broad goals: testing a novel approach to algebra that focused on helping students learn skills for building and using mathematical models to solve real everyday problems, and developing techniques for the successful transfer of technology into the classroom. Overall, the technology transfer was effective, insofar as the computers functioned as a standard component of daily lesson activities, typically used by each student twice a week. Thus, the technology transfer model, whereby a team of researchers and teachers provide computer-integrated lesson plans and course materials, proved successful in enabling substantial computer use for new activities. However, teacher comfort with the technology varied greatly between the two teachers. Increased training that includes sufficient time for teachers to become familiar with computer capabilities and assignments and the operation of the machines themselves is necessary to help teachers become more comfortable with the new teaching tool.

The novel algebra curriculum was intended to teach students to acquire several skills, including identifying the variables in a real-world situation; describing the covariation of the values of dependent variables; recognizing and interpreting multiple representations of information (tabular, graphical, and equational); constructing qualitative and quantitative models of situations; solving problems on several levels (specifying solution goals and operations and performing symbol manipulations); and interpreting solutions in terms of the original problem. The curriculum achieved mixed effectiveness. In general, students were more successful at creating and using qualitative models than quantitative models. In the qualitative modeling section of the curriculum, over two-thirds of the students could successfully identify the variables, build qualitative models, gather data to explore relationships, interpret tabular and graphical representation, and describe relationships. On these innovative course items, they outperformed the algebra students, a significant result given that the Math A students' basic math abilities were significantly less than their peers in the algebra classes.

Students were less successful at constructing quantitative models and equation solving. The class unit tests indicate that only about a quarter of the students could specify the three operators necessary to transform a word graph to a word equation; about half of the students could specify the appropriate goals for equation-solving steps; about a third could specify the appropriate operation; 45 percent could perform the symbol manipulations to solve problems arising from real-world situations; and 24 percent could perform the symbol manipulations to solve abstract problems.

Four factors appear to have influenced our results: (1) The target population we selected had limited mathematical skills. Though final outcomes were low, we were encouraged by the substantial gains in skills made by students during the study. (2) The teachers had difficulty in adopting a new approach to algebra and new roles in the classroom despite extensive involvement in the course development. Our class observations often noted the teachers' difficulties in assimilating new theoretical goals. For example, the teachers tended to teach the goals of particular steps in problem solving as processes, emphasizing what to do, not why to do it. The curriculum also introduced challenging roles for the teachers including facilitating student computer activities and acting as computer trouble shooter. The teachers displayed varying levels of comfort with computer activities indicating again the importance of training experiences to acquaint teachers with innovative approaches to mathematics and new course tools and materials. (3) A third factor contributing to our results was the slowness of the computers and frequent bugs causing computer crashes. Students' time at the computers was restricted and the computers did not entirely assume the teaching role they were expected to have. (4) Finally, to understand our results, we looked more closely at the equation-solving activities in the curriculum, which were the activities that students found most difficult. We had expected that the familiarity with problem-solving goals and operations would produce an easy transition to direct symbol manipulation. Students had only one computer assignment to practice direct symbol manipulation. Students' poor equation-solving skills may indicate that more practice in direct symbol manipulation is desirable or that the top-down approach to problem solving was not effective in promoting acquisition of equation-solving skills. Students may require more familiarity with symbol manipulation before they are able to specify problem-solving goals. Though their overall posttest equation-solving scores were low, students did make significant gains in their skills, particularly in nonabstract equation solving. Students were twice as successful in solving equations that arose from real-world situations as they were in solving exactly the same type of equation presented abstractly. We speculate that situating equations allow students to draw on their intuitive and experiential resources and is more intrinsically motivating.

NEXT STEPS

Our next step is an expanded version of the curriculum module described above to include material on statistics. We will use the technology transfer model successfully fielded in this pilot but will devote more attention to teacher training and teacher assimilation of course goals.

ACKNOWLEDGMENTS

We are indebted to the adventurous teachers, students, and administrators of Santa Monica High School for their cooperation and patience in allowing us to pilot the innovative approach to algebra reported here. In particular, we would like to thank Catherine Baxter, who spent countless hours helping to develop and edit the courseware and transform it into a teachable curricular unit.

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1. INTRODUCTION

The crisis in education articulated in *A Nation At Risk* (1983) and the Carnegie report (1986) has focused extensively on the poor mathematics performance of American students. This concern is reflected in the 1986 National Assessment of Educational Progress (Dossey et al., 1988) and in the complaints of employers that workers do not have appropriate skills to contribute successfully to a technology-dominated workplace (Romberg, 1987). In response to the call for reform, the national council of teachers of mathematics (NCTM) proposed a new set of standards for K-12 mathematics instruction (Romberg, 1987). It suggests a variety of fundamental content changes for mathematics education. At the most general level, the NCTM standards emphasize the need for students to learn *higher-order thinking* and *problem-solving skills*. Conversely, the standards deemphasize the rote or algorithmic manipulation of mathematical symbols, arguing both that the rapidly changing workplace demands the acquisition of flexible problem-solving skills and that computers should take over the computation of routine mathematical results.

The NCTM standards lead to several specific questions concerning the content of mathematics courses and the role of computers in the classroom. What problem-solving and higher-order thinking skills should replace traditional ones in the classroom? What techniques should be used to teach these new skills? How should computers be introduced into these courses to support students' thinking by doing routine mathematical computations? Should computers be used simply as a problem-solving resource or should they take a more active role in teaching students?

The RAND Algebra Tutor Project has been working for the last four years on developing and piloting intelligent computer tools for first-year algebra students. Our work began with the development of an intelligent tutoring system (ITS) to help students learn symbol manipulation skills for solving problems in basic algebra (McArthur, 1987; McArthur et al., 1987; McArthur et al., 1988; McArthur and Stasz, 1990). Consistent with the NCTM standards, subsequent versions of our ITS have focused on helping students learn more "strategic" reasoning skills in algebra, leaving the "tactics" of symbol manipulation to the computer. For example, one version decomposes solving equations into several levels of decisionmaking and requires students to specify only the goals or operations needed to solve an equation (e.g., "collect terms" or "add 5 to both sides") whereas the ITS executes the lower-level symbol manipulation that implements these instructions.

We have found that in partnerships where much of the underlying mathematical manipulation is being handled by the computer, students have opportunities to focus on important aspects of problem solving (e.g., defining variables, describing relationships among variables, developing strategies for problem solution) that are often difficult to include in traditional classroom instruction. However, we have also found that shifting the emphasis of our algebra tutor from more conventional content and pedagogy to such partnerships leads to problems integrating the technology into the classroom. Like other tutoring systems (Schoenfeld and Verban, 1988, Strudler and Gall, 1988), the algebra tutor suffered from implementation difficulties (see Robyn et al., 1989; Stasz et al., 1989).

Inadequate training, new instructional, pedagogical, and organizational goals, and logistical problems all contributed to the difficulty of effectively implementing computer-assisted instruction. We found that the classroom culture placed priority on covering a given set of topics in a certain amount of time. Topics were taught didactically, and the pace of the class was fast and textbook driven. The teacher and students were intent on covering material relevant to the textbook and traditional test items. Tests emphasized skill in abstract symbol manipulation. In addition, the teacher did not monitor students' work at the computer, limiting her coaching to students engaged in noncomputer activity. As a result, the nontraditional computer approach that we introduced, emphasizing higher-level cognitive skills of problem-solving strategy, was not evaluated as part of the coursework and hence had little value for students or teachers.

Given these fairly typical classroom features (Tobin, 1987), it has become clear that implementing a technology-based course requires more than just the development of computer tools that focus on novel mathematical skills. Successful implementation demands the development of a *full curriculum*, comprising a set of support materials that permit the classroom culture to change to take advantage of the tools and new curriculum content. Supports include lessons that directly integrate computer activities into the coursework, training for teachers in managing the curriculum and technology, and a class setting freed from having to teach to traditional tests and from traditional instructional sequences. We believe these materials are as important as content and technology changes in improving the quality of learning of mathematics in our classrooms.

In this Note we describe the curriculum module for beginning algebra that we developed to meet the challenges discussed above. Our curriculum by no means addresses all the problems associated with the reform of mathematics education and with the use of computers in the classroom. Rather, it should be viewed as a prototype computer-based course that attempts to address, in an *integrated* rather than *piecemeal* way, some problems of mathematics curriculum content reform, the novel use of computers in the classroom, and implementation problems that arise from these content and technology changes.

In Section 2 we describe the development and content of the curriculum. We discuss our overall approach to curriculum development, the specific novel mathematical topics we targeted, as well as the design of the computer software and teacher and student materials. Section 3 discusses the implementation of the module, student and teacher training procedures, and our assessment procedures. Section 4 evaluates the curriculum and its implementation, and we conclude in Section 5 with a discussion of the study results and proposed modifications of the curriculum.

2. THE CURRICULUM

The curriculum we developed departed from traditional algebra instruction in three ways: The pedagogy stressed new techniques situating learning in real-life problems, introduced a conceptual framework before local skills practice, and used inductive instruction and cooperative learning; the course content included innovative topics—modeling, covariate relationships, and problem-solving strategies; and the teaching tools included daily access to an intelligent tutoring system. Curriculum materials included teacher lesson plans, parallel computer and noncomputer activities for students, and evaluation instruments. In this section we elaborate on these aspects of the curriculum beginning with our overall approach to curriculum development.

APPROACH TO CURRICULUM DEVELOPMENT

Designing a new curriculum that encompasses the features outlined above is a complex undertaking that requires different kinds of expertise. Although computer scientists and programmers can design computer-based learning tools, they may have little knowledge of how to use computers in classrooms. To ensure the viability of computer tools and, more generally, to develop a curriculum that would realistically transfer from the context of development to the context of actual use, we formed a multidisciplinary curriculum development team. This team—composed of computer scientists, cognitive and educational psychologists, and implementation and curriculum experts—determined the cognitive goals for the course and the basic course outline. The proposed outline was then reviewed by several teachers, two of whom joined the development team. Two subgroups formed to develop the course package; one group concentrated on computer software and the other on teacher lesson plans. Throughout the three-month development period both teams met frequently to ensure that software design meshed with other curriculum activities. The highly interactive process led to frequent adjustments and modifications as materials were actually developed and tested with a few students in a laboratory setting.

Figure 1 illustrates how we conceived the design task. Central to our planning were *content goals*—the specific kinds of knowledge and skills we wanted students to learn—and *pedagogical goals*—the ways of learning and teaching course material we wished to promote. These informed the development of software tools and lesson plans and dictated the evaluation measures. Underlying this framework was a consistent concern that the end

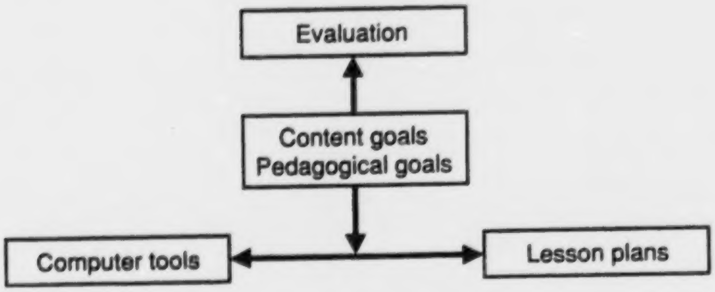

Figure 1—Design Tasks

result be viable in the classroom. Thus, we also considered such issues as the classroom culture and teacher and student comfort with the curriculum.

CONTENT GOALS

The course subject matter centered on teaching students various *model-building* skills, similar to those noted in NCTM and other related research (e.g., Fey et al., 1984; Thompson and Rathmell, 1988; Usiskin, 1985). By model-building skills we mean the ability to see how formal mathematical objects can relate to the real world, for example, how equations can provide powerful descriptions, or models, of the interactions among real-world objects or properties. Model building includes several skills rarely taught in traditional classrooms:

- *Formulation.* In a given real-world situation, formulation involves determining which objects or properties should be represented as mathematical objects or variables. For example, in modeling the finances of a car wash to raise money to send the marching band to a distant competition, one might decide that properties such as the number of cars and the amount one has to spend on supplies should be variables in an equation that can be used to predict profit or loss.
- *Data gathering, representation, and building qualitative models.* This refers to a set of related skills for collecting and examining information about values of properties selected during formulation. The intent of these activities is to understand empirical patterns of co-occurrence among variables of situations, leading students to an intuitive or qualitative model of the situation. For example, in the car wash situation, one might record tables of values and graphs showing how profit appears to covary with number of cars washed. The ability to

use multiple representations of relationships is key to successful comprehension of patterns in data (Kaput, 1989).

- *Translation of qualitative models into quantitative ones.* Translation involves positing an equational representation that captures the observed patterns of interrelations of the formulated situational properties. For example, in the car wash situation, translation may involve postulating an equation that relates the variable for profit to variables representing the amount spent on supplies, the fee charged per car, and so on. This skill is related to hypothesis generation.
- *Inferencing or prediction* involves answering questions about the situation at issue, either by manipulating equational representations or using less formal guess-and-test or extrapolation techniques that use graphs and tables. For example, given a problem in which one must infer the required duration of exercise to achieve a given weight loss, one can use an equational model for the situation or perhaps consult graphs and tables of values. This skill is related to hypothesis testing.

PEDAGOGICAL GOALS

Our pedagogical approach also introduced some departures from traditional algebra instruction.

- *Learning about mathematical tools in the context of natural situations.* One general principle we adhered to was that students needed to learn algebra as a tool for solving problems in everyday life. Problems should be grounded in natural situations and problem solving should reflect the cooperative arrangements frequently found in the real world. High school algebra coursework often focuses on abstract principles (e.g., the commutative rule) and algorithms for solving abstract problems (e.g., the FOIL technique for multiplying factors). Word problems are often thinly disguised wrappers for equally abstract problems that seem to have little relevance to students' concerns. The new NCTM math standards, other research (e.g., Fey et al., 1984; Schoenfeld, 1985; Collins, Brown, and Newman, 1989), and our own experience underscore the value of an approach grounded more in the real world. All coursework problems arose from familiar situations. We selected topics that were relevant to the lives of 1980s teenagers, such as after school jobs, rock concerts, renting limousines for the prom, and pollution of the local bay.

- *Global before local skills.* A second pedagogical principle underlying the curriculum was that students should be introduced to more global skills before learning the skills for local operations (e.g., Brown, Collins and Duguid, 1988). Thus, strategic goals for problem solving were introduced before specific symbol manipulation techniques or algorithms. For example, students were taught to specify the goals for each step in an equation solution before they were taught the mathematics for accomplishing the goal. As discussed below, the computer supported this approach by automatically doing the symbol manipulation.
- *Learning through inquiry as well as didactic presentation of material.* Whenever possible, material was introduced in ways that fostered inductive instruction rather than by didactic presentation. The computer activity on covariation, for example, required that students input different values for several variables and then note the effect on an output variable. Students described how variables might affect each other (e.g., positively, inversely) on an accompanying worksheet. In this way, students were often able to use their strong informal skills for describing relationships to form intuitive mathematical models of relationships among variables before learning how to formalize them using equations and how to manipulate the equations.
- *Group collaborative learning.* When appropriate, students worked cooperatively so that they could acquire skills in cooperative problem solving and take advantage of the opportunities for more effective discussion and exploration offered by this type of interaction (Slavin, 1986). Seatwork activities, such as an assignment to construct a qualitative model for a T-shirt sale and build a table to explore the effects of changes in the amounts of the variables, were often carried out in groups of four students. All computer work was conducted by pairs of students.

THE COMPUTER TOOLS

Different computer environments, including several common tools, were built to help students engage in each of the main activities involved in model building, formulation, data gathering and representation, and translation and inferencing using equations. More generally, these activities reflect the basic organization of the course materials and assignments. Early lessons and assignments focused on helping students determine which aspects of verbal situations represented the mathematical objects or variables of interest.

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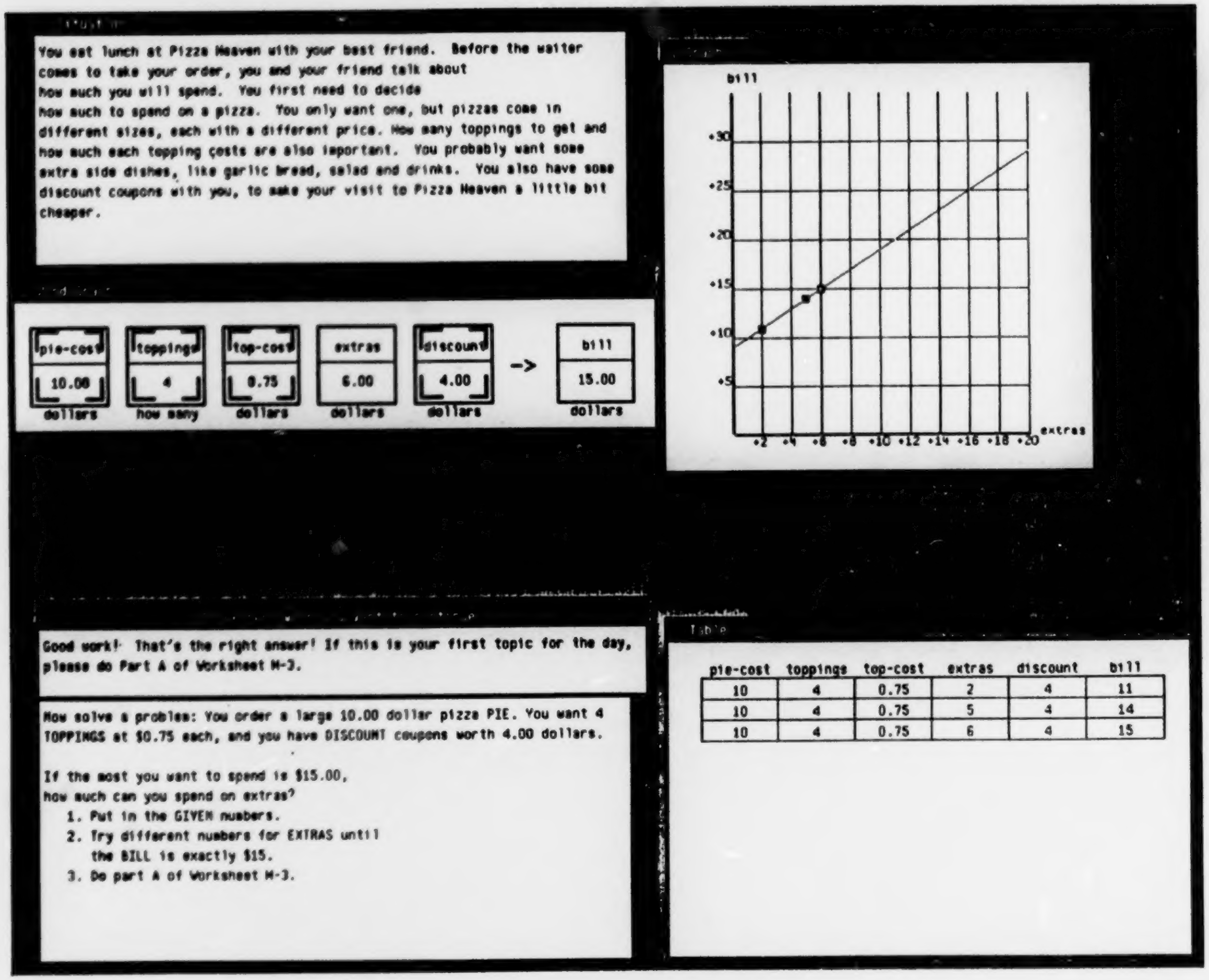
Then the students acquired skills for building input-output flow models of abstracted situations. Following this, students explored covariate relationships among variables by changing values of variables and observing the propagated effect on other variable values. Once students had an opportunity to develop a qualitative understanding of the relationships among variables, they were coached in expressing algebraic or quantitative models of the relationships. Finally, the students learned to manipulate formalized equations to answer a variety of specific questions about the situation. Here the embedded algebra ITS provided several levels of coaching. Below, we discuss computer environments for each of these main activities in the overall lesson plan.

Computer Environment for Formulation and Development of Qualitative Models

Curriculum software was implemented in Franz Lisp on Sun Microsystems workstations (see McArthur et al., 1987; McArthur and Stasz, 1990, for a more complete description of the tutor architecture and functionality). For each assignment, students can select a situation from a menu of options (e.g., video store job, pizza party, weight estimation, boating). The tutor keeps track of which assignments and situations students have completed. The first time a student selects a situation he or she must always complete a model-building activity that specifies component variables. Subsequently, the computer will automatically rebuild the models, graphs, and tables that the student might have constructed in previous sessions. Figure 2 shows the model-building software interface for an early assignment where students are interested in determining the variables in a situation and gathering data to form a qualitative model of variable relationships. The situation to be modeled is described verbally in the "situation window" at the upper left. Notice that the situation description is not a verbal problem but an informal description of variable relationships. Later, the student will answer several specific questions based on this situation. Below the situation description is a "word graph" where the student inserts variables he or she has identified from a menu. The variables relate the dependent variable to various independent factors. Using the word graph, the student can input values for selected variables. The tutor then computes the appropriate values for other variables, based on an underlying equation that relates the variables.

In the early assignment shown in Fig. 2, the student's goal is to use guess-and-test strategies to understand co-occurrence relationships among variables and to answer specific questions. Students insert values into the word graph boxes and the environment provides several tools in which the data are automatically plotted to help the student gather and organize the data. These tools include tables of values (lower right) and cartesian graphs



You eat lunch at Pizza Heaven with your best friend. Before the waiter comes to take your order, you and your friend talk about how much you will spend. You first need to decide how much to spend on a pizza. You only want one, but pizzas come in different sizes, each with a different price. How many toppings to get and how much each topping costs are also important. You probably want some extra side dishes, like garlic bread, salad and drinks. You also have some discount coupons with you, to make your visit to Pizza Heaven a little bit cheaper.

pie-cost	toppings	top-cost	extras	discount	->	bill
10.00	4	0.75	6.00	4.00		15.00
dollars	how many	dollars	dollars	dollars		dollars

Good work! That's the right answer! If this is your first topic for the day, please do Part A of Worksheet M-3.

Now solve a problem: You order a large 10.00 dollar pizza PIE. You want 4 TOPPINGS at $0.75 each, and you have DISCOUNT coupons worth 4.00 dollars.

If the most you want to spend is $15.00,
how much can you spend on extras?

1. Put in the GIVEN numbers.
2. Try different numbers for EXTRAS until the BILL is exactly $15.
3. Do part A of Worksheet M-3.

pie-cost	toppings	top-cost	extras	discount	bill
10	4	0.75	2	4	11
10	4	0.75	5	4	14
10	4	0.75	6	4	15

Fig. 2—Modeling Environment

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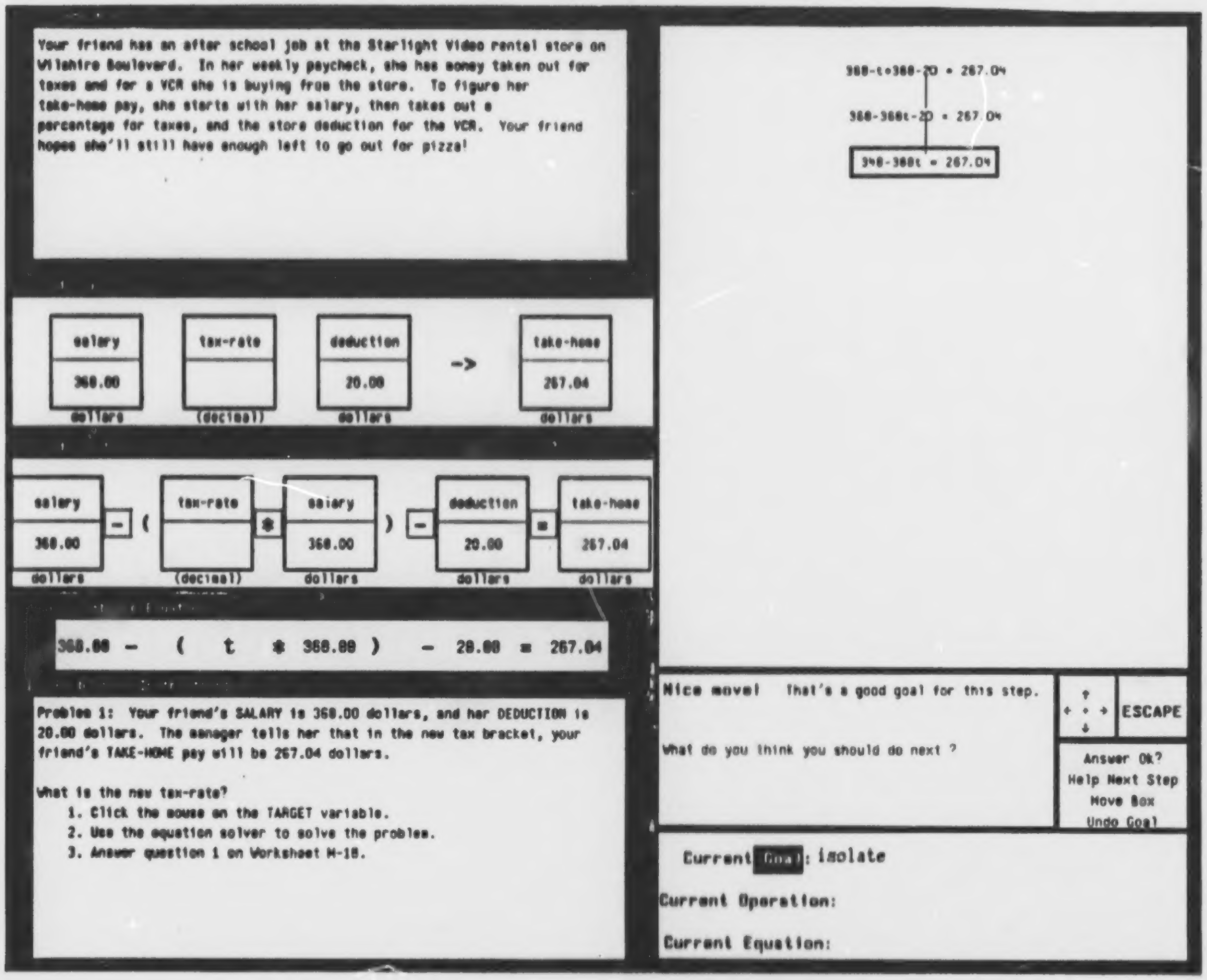
(upper right). In addition, the student is guided in data gathering by specific problems (lower left). After students have a chance to freely explore varying values for variables, the problems generally constrain the student to explore covariation of pairs of variables. Here, students can vary the amount spent on "extras" (side dishes) while the pie cost, toppings cost, number of toppings, and discount coupon value stay the same. The data gathered in this "constrained exploration" permit the student to observe a positive linear relationship between the amount spent on the "extras" and the total bill. In addition to answering specific questions, these early assignments were to help students develop an intuitive or qualitative model of situations which in later sessions would be translated into a more quantitative, equational representation.

Computer Environment for Translation and Answering Specific Questions

In later assignments students could choose to work with the same situations they informally investigated on a previous day. As Fig. 3 shows, many computer tools (e.g., the situation description and word graph) are retained in later sessions. In addition, several tools appear to help students construct operators (*, /, +, -) and equations for the situation and to answer specific problems. Equations are first represented as "word equations" (below the word graph in Fig. 3), which look like word graphs with operators connecting the "boxes" that represent variables. Next, symbolic equations are computed from word equations by substituting letters for variable boxes (see Fig. 3). Subsequently, students are given a series of problems about the topic. The students can then use their equational models to generate answers. It is at this point that the equation-solving ITS comes into play, coaching the student through the manipulations required to make the desired inference.

The algebra ITS embeds several kinds of knowledge that help it tutor students in solving equations. It knows the subject—the ITS is an expert in solving problems in simple symbolic algebra; it knows the student—it can infer limited diagnostic models of the students' knowledge of algebra; and it knows something about teaching—it includes some simple rules of pedagogy that help it determine what information to provide and when to provide it. The ITS's knowledge of algebra is *articulate* in a way that enables students to learn the different skills for solving equations in a systematic and top-down fashion. When solving an algebra problem, the ITS reasons separately about what *goal* to pursue (e.g., isolating the unknown), what *operation* implements the goal (e.g., adding 5 to both sides of the equation), and how to manipulate the symbols to accomplish an operation. Because the ITS algebra knowledge is articulate, we can structure specific student assignments to focus on each of these different levels of knowledge. In Fig. 3, for example, the student is solving



Your friend has an after school job at the Starlight Video rental store on Wilshire Boulevard. In her weekly paycheck, she has money taken out for taxes and for a VCR she is buying from the store. To figure her take-home pay, she starts with her salary, then takes out a percentage for taxes, and the store deduction for the VCR. Your friend hopes she'll still have enough left to go out for pizza!

360.00 - (t * 360.00) - 20.00 = 267.04

Problem 1: Your friend's SALARY is 360.00 dollars, and her DEDUCTION is 20.00 dollars. The manager tells her that in the new tax bracket, your friend's TAKE-HOME pay will be 267.04 dollars.

What is the new tax-rate?

1. Click the mouse on the TARGET variable.
2. Use the equation solver to solve the problem.
3. Answer question 1 on Worksheet M-18.

360-t*360-20 = 267.04

360-360t-20 = 267.04

340-360t = 267.04

Nice move! That's a good goal for this step.

What do you think you should do next ?

Current Goal: isolate

Current Operation:

Current Equation:

salary
360.00
dollars
tax-rate
(decimal)
deduction
20.00
dollars
->
take-home
267.04
dollars
salary
360.00
dollars
-
(
tax-rate
(decimal)
*
salary
360.00
dollars
)
-
deduction
20.00
dollars
=
take-home
267.04
dollars
↑
← • →
↓
ESCAPE
Answer Ok?
Help Next Step
Move Box
Undo Goal

Fig. 3—Equation-Solving Environment

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the symbolic equation developed to reflect the problem posed in the problem window at the lower left. Once developed, the symbolic equation automatically appears in the equation-solver windows on the right. In this example, the student is working at the goal level, specifying appropriate goals for each step of the equation. Goals include SIMPLIFYING the equation, ISOLATING the variable, COLLECTING terms, REDUCING and DISTRIBUTING. The ITS, in this example, is prompting the student for the goal for the next step in solving the equation (lower-right window) and, if the goal the student selects can be executed, the ITS will automatically implement the operation and manipulation necessary to carry out the goal. In this way, learning and problem solving involve *role sharing* between the computer and student. The current step in solving the equation is indicated by a box.

Student-computer role *profiles* are defined for each assignment and determine what work will be done by student and by machine. This permits flexible cooperation. Consistent with our top-down pedagogical philosophy, in equation-solving assignments the standard role profile dictates that the student will first select only goals and that the ITS will supply operations and manipulations. Later, students become responsible for operations, and in the final assignments students do goals and manipulation. Thus, only at the end of the course will students do the "symbol crunching" that is central to most introductory algebra courses.

The algebra ITS also embeds several other pedagogical rules. In the model-building activities, students can select correct variables, operators, or symbols. If they make two errors in a row, the computer supplies the correct term. In the data collection/covariation activities, students continue collecting data until they feel they have the right answer. The activity is open-ended, allowing for unlimited exploration, and no coaching is provided. In the equation-solving environment, students have access to the equation solver by asking for "help." The tutor will specify the conditions for that step in the equation and the goal, operation, or symbol manipulation as appropriate for the problem. If further help is requested, the tutor will illustrate how to perform the step. Students can also ask the tutor to check whether any equation in the solution tree is correct.

TEACHER AND STUDENT MATERIALS

Teacher Materials

To provide support for teachers and insure that the course material would be taught as intended, we developed specific lesson plans (see Appendix A for full set of curriculum materials). Introductory material in the curriculum package describes the course goals and the pedagogical techniques to be used. A typical lesson begins with a review of a previous topic. The teacher next introduces the topic of the day and provides practice exercises for the topic that are completed by the class and then reviewed with the teacher. Then individual (or small group) practice is conducted either through noncomputer or computer assignments. Each lesson plan contains the goals for the lesson and an almost scripted set of activities to carry out, including board examples for the teacher to use. Specific lessons are included for introducing the various computer environments. Teachers also have available printed review sheets—used for warm-ups and practice activities, noncomputer assignment sheets, computer assignment sheets, and vugraphs of materials for whole class review. The lesson plans, computer assignments, and tests for each of the two units were developed by the research staff with teacher input, and most of the review assignments and noncomputer assignments were developed by the teachers with staff input. We hoped by this process to thoroughly acquaint the teachers with the course goals, pedagogy, and materials.

Student Materials

Twenty assignments were prepared for students—eight computer assignments and 12 noncomputer assignments. Students received booklets that contained all of their assignment worksheets. Each computer assignment was accompanied by a brief worksheet on which students could record the situations selected and the results of their explorations and note the methods they had used. These worksheets were designed primarily to provide hardcopy records of student computer work for teacher use and to make computer and noncomputer assignments more alike. Whenever possible, noncomputer assignments parallel computer assignments but do not necessarily share the same features. For example, in exploring covariation on the computer, an underlying equation permits the tutor to compute a missing value. In paper assignments, the equation is explicit and is computed by students. In equation solving with the tutor, the student can solve an equation by specifying goals while the computer handles all the symbol manipulation. In noncomputer assignments, students can specify a goal for a specific solution step, but until they can perform the symbol manipulation corresponding to the goal, they cannot move the solution ahead to the next step.

Noncomputer assignments provide practice in (i) specifying goals and operations for a given step in a solution; (ii) completing the symbol manipulation for the specified goal/or operation; (iii) solving an entire problem by specifying the goals and completing the symbol manipulations; and (iv) symbol manipulation (without goal specification). Generally, the opening problems in a noncomputer assignment are situation-based (like the computer problems), whereas the remainder are more abstract and provide students computational practice.

A TYPICAL LESSON

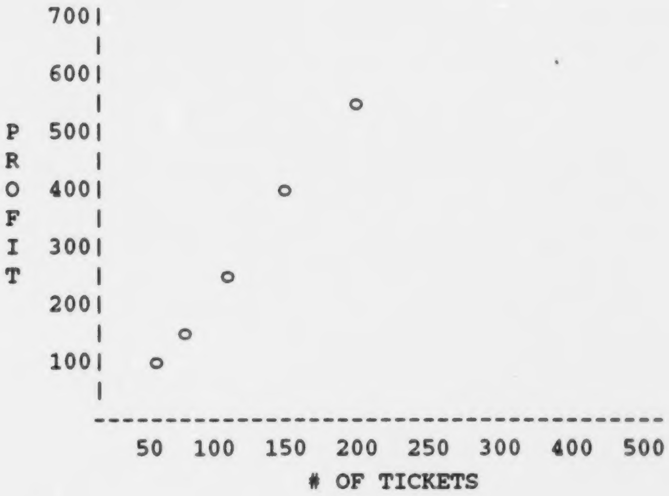
To see how all the different pieces of the curriculum package fit together, imagine, for a moment, a typical classroom day during the second week of the curriculum. Students arrive and find the day's activities listed on the board: Warm-up; Introduction to graphing; Worksheet #4 or Computer Assignment #3. With a bit of noise, some corn chips snatched out of a bag, and a search for pencils, the class settles down to do the warm-up review exercise while the teacher checks attendance. A word graph on the board represents the profit to be made from a Black Student Union/Mexican Association dance:

```
# of tickets   ticket price  expenses ==> profit
```

Below the graph is a table in which price and expenses are held constant. Students are instructed to complete the table, calculating the profit based on different numbers of tickets sold:

# of tickets	*	ticket price	-	expenses	=	profit
40		$3.00		$75.00		$45.00
50		3.00		75.00		
75		3.00		75.00		
100		3.00		75.00		
150		3.00		75.00		
200		3.00		75.00		

The teacher asks questions about the table to illustrate basic concepts: "As you increase the number of tickets, what happens to the profit?" (covariation); and, "As the number of tickets increases what happens to expenses?" (effects when one variable is held constant). Moving on to the topic of the day, the teacher draws a graph relating profit to tickets and with student help plots points from the table:



The teacher asks how much money could be made if 250 tickets were sold and how many tickets would have to be sold for a profit of $675.00. Students answer the questions and the teacher asks them to explain how they arrived at the answer, thus, forcing them to articulate their reasoning processes.

The teacher uses the noncomputer assignment sheet to continue the explanation. On an overhead projector, she displays the first section of this worksheet, which contains a situation discussing blood pressure as a function of age, a table showing several age inputs and resultant average blood pressures, and a graph plotting the age inputs against blood pressure (see the Student Materials, Assignment M4, p. 1, in Appendix A). The assignment continues with questions aimed to guide students through reading the table and the graph. The teacher goes through a few questions with the whole group to check their understanding. The assignment contains an additional situation (with relevant tables, graphs, and questions) designed to help students learn to use tables and graphs to solve problems.

For the second half of the period, half of the students complete work on the noncomputer assignment and the other half work in preassigned pairs on the computer assignment (the following day, students will switch assignments). At the computers, students login and select a situation to work on. Anna and Sara select Pizza Heaven, a situation they enjoyed working on last time. Since they have worked on this situation before, the computer displays the word graph they made previously and then the first problem of today's assignment: varying the cost of extra side dishes to achieve a total bill of $15.00 (see Fig. 2, which shows how values are input into word graphs). The girls hesitate for a moment and wonder what to do first. The teacher, circulating through the room, notices their puzzled looks, and reminds them to look at the instruction window. Working as a team, Anna clicks the mouse on each word graph variable box, and Sara types in the amount given in the problem for each variable—the cost of the pizza pie, the number of toppings, the cost per topping, and the amount of a discount coupon. The girls discuss the situation and decide that all they want extra is drinks, so they input $2.00 for "extras." The computer computes the total bill, and the girls find that they are still $4.00 under the $15.00 total. All of the data input into the word graph is recorded in a table, and a graph of the amount input for "extras" against the bill total is also displayed. The girls continue inputting different amounts for "extras" until they achieve the $15.00 total bill. The computer then directs them to complete the first part of their worksheet for this assignment, M3 (see Student Materials, Assignment M3, part A, in Appendix A). The worksheet asks students to record the problem and their answer. Then it asks how they arrived at the correct answer and to explain to someone else a technique for getting the answer.

When Anna and Sara complete their worksheet, they continue with the next computer problem, which requires them to use a graph to determine the price of pizza for a given bill value if all other variables are held constant. The girls click on the graph's "bill" axis at the location that corresponds to the value for the bill, and the computer visually computes the corresponding value on the "pizza" axis. The computer then asks them to record their exploration on a worksheet, which also asks what "pizza" input would produce a very high "bill" output. Since there is still ten minutes left in the period, the girls select a second situation—"Heal-the-Bay"—and complete similar problems. When they are finished, they logout and return to their seats.

3. IMPLEMENTATION

We implemented the novel curriculum over five weeks in the fall of 1988 in a local high school. In this section, we describe the teachers and students who participated in the study, teacher and student training, and classroom operations and data-gathering procedures used in this study.

THE STUDY PARTICIPANTS

Teachers

The curriculum module was taught by two teachers who serve as consultants to the RAND Algebra Tutor projects and were members of the curriculum development team. Teacher A is a young woman with several years of secondary school math instruction experience. She is interested in curricular innovation and the use of technology. She is a member of the UCLA California Mathematics Framework Project, helped launch a computer program at a Los Angeles Unified School District magnet school, and was designated by the high school to design an innovative "Math A" course curriculum. As a member of the project's curriculum development team, she had responsibility for developing many of the noncomputer assignments and extensively reviewed the teacher lesson plans and software specifications.

Teacher B is a senior member of the same high school faculty and has over 20 years of mathematics teaching experience. She has been actively involved in curricular reform and designed a statistics course for the high school. She has made presentations at national conferences on such topics as "Liberal Arts Math," "Integrating Computers in Math Courses," "Finite Math Topics," and "Statistics and Probability in the Traditional Course of Study." In recognition of her expertise, she was selected as a Woodrow Wilson Fellow in Statistics at Princeton and received her school district's outstanding faculty award. Teacher B's classes participated in both of the Algebra Tutor Project's previous field studies (see McArthur and Stasz, 1990; Robyn et al., 1989; Stasz et al., 1989). During the curriculum development phase of this study, Teacher B helped develop and review materials but took a less active role in the project than Teacher A.

Students

Participating students were enrolled in either Math A or regular algebra classes. Math A is the first year of a new two-year course implementing the recent California Math Framework (California State Department of Education, 1985). It features several discrete units—probability, symmetry, statistics, algebra, and geometry. The course is intended to expose students to enough algebra and geometry to enable them to continue in a college-focused academic plan. The course targets less-successful students who typically are low achievers in an algebra class. Our curriculum module comprised the introductory algebra unit in the Math A course.

All four Math A classes participated in the study. Two sections were taught by each of the teachers. Parent consent was obtained for participation in the data-collection activities. The average class size was about 32, although actual daily attendance was about 28 students. To provide comparative information, four regular algebra classes were asked to participate in the data-collection activities. Classes were selected to match as closely as possible the periods of the Math A classes (one class time could not be matched), and each class was taught by a different teacher to avoid a teacher effect. All of these classes progressed at about the same pace through the same textbook. At the time of the study, the algebra students were working on linear equations and factoring.

TEACHER AND STUDENT TRAINING

Since both pilot teachers were members of the curriculum development team, formal teacher training was required only for the computer component of the course. The software was available for teacher review as it was developed. During two laboratory sessions, materials designed to train students to use the tutor for the modeling and equation construction environments were demonstrated by staff members. The teachers then coached students through the relevant computer assignments. The equation-solving software was not available for laboratory piloting but was available to the teachers shortly before it was used in the classroom.

The two teachers differed dramatically in the amount of time they spent learning about the computer software and hardware. The difference seemed to be a function of the teachers' own interests, the constraints of teaching and class preparation duties, and other professional responsibilities. Teacher B spent approximately five hours in computer training (the time spent at the two lab sessions) and Teacher A spent about ten hours (including additional lab sessions with students and individual exposure), and about 20 additional hours (approximately an hour during each day of the implementation) debugging software.

The teachers received little preliminary training on troubleshooting computer problems; project staff attended every class and were expected to handle any emergencies. Just before the curriculum module began, Teacher B decided that she did not want to be responsible for student work at the computer, so project staff took over this task. The teacher felt too pressured to monitor the computer work in addition to teaching new course material. Teacher A, on the other hand, decided to handle all class activities with a minimal amount of help from project staff. Thus, she received additional instruction about troubleshooting computer problems.

Student training materials enabled the teacher to instruct the whole class at once. Using an overhead projector and transparencies of the Sun workstation operation, the teacher walked the students through login procedures and the different computer environments and activities before the students actually sat down at the computers (see Teacher Lesson Plans, pp. 7–9 and pp. 24–25, in Appendix A). Teacher A spent about an hour introducing the computer and various environments and Teacher B spent about 20 minutes.

CLASSROOM OPERATIONS AND DATA-GATHERING PROCEDURES

Seven Sun 3/50 workstations were installed in the Math A classroom. Project staff were present in every class to handle any computer operational problems that arose. The curriculum was designed to span 13 days, but teachers were not required to adhere to a rigid schedule. Teachers were encouraged to set their own pace and they taught the course in 20 days. Class observation data indicate that teachers spent about ten days on the modeling portion of the curriculum and about 15 days on equation solving; this time was proportionately equal to the original schedule.

We used multiple methods to assess the curriculum, including mathematics and algebra achievement tests, two unit tests, background and attitude questionnaires, classroom observation instruments, and computer records. (See McArthur and Stasz, 1990, and Stasz et al., 1989, for further description of our overall approach to data collection). These instruments are included as Appendix B. Students took an arithmetic skills test to measure arithmetic ability and a pretest and posttest that assessed student experience with and attitudes toward computers and mathematics and their problem-solving skills. The skills test included traditional abstract equation-solving problems, as well as novel problems featured in the curriculum (e.g., identifying variables in a natural situation, constructing an equation to represent a natural situation, solving problems related to the situation, and specifying problem-solving strategies). The set of background questions (e.g., gender, age, grade in school, past mathematics courses, feelings about algebra, computer experience) have been tested and refined over the course of several Algebra Tutor projects. The attitude questions, also used in our previous research, were developed by the research team or adopted from other studies (e.g., Schoenfeld, 1985; Carpenter et al., 1980). These

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instruments were also given to four Algebra 1 classes. The pretest was administered just after the algebra students completed lessons on linear equations. The posttest, given five weeks later, followed lessons on factoring. Thus, algebra students' knowledge of equation solving at pretest was similar to the Math A students' knowledge at posttest.

Math A students also completed two unit tests—one on identification of variables and covariation and the other on equation solving. These are described more fully in Sec. 4. The posttest for Math A students also included student ratings of the course. Computer records automatically recorded student time on the computers and assignments completed. To assess implementation of the novel curriculum, project staff kept daily class observation records of the topics presented, the effectiveness of the teacher's presentation, student experiences with the class and computer activities, and general computer operation. These records also included teacher comments about the course and general student progress.

4. EVALUATION OF THE CURRICULUM

In this section we discuss our overall approach to evaluating the curriculum and present student outcomes. We examine these outcomes with respect to intended and actual implementation of the curriculum, as indicated in classroom observations of teaching and learning. Finally, we examine students' evaluation of the course and assess the curriculum's limitations.

EVALUATION DESIGN

The evaluator seeking to determine the effectiveness of any instructional program has a wide range of alternative designs from which to choose (e.g., Campbell and Stanley, 1966; Cook and Campbell, 1979). Choosing which is best depends on a number of factors, including the types of decisions (and decisionmakers) on which the evaluation focuses and the feasibility of implementing the design (Shavelson et al., 1986).

Generally, two questions are raised in most curriculum evaluations: How much do students gain in knowledge, skills, and attitudes from the instructional intervention? Is what they learn the same as what students learn in traditional courses that are intended to teach the same subject matter (Shavelson et al., 1986)? In this study, we weight the question of knowledge gain as more important than the question of equivalence of outcomes. We believe this focus appropriate for a curriculum development project which is innovative in content, pedagogical approach, and supporting instructional materials. Thus, our evaluation approach is more formative than summative (see Flagg, 1990, for approaches to formative evaluation of educational technologies). The aim is to pilot test a prototype curriculum and gather data to understand which aspects of the curriculum were successful and which need revision.

A third evaluation question, less often addressed, concerns implementation: What factors in the implementation process influence the program's success or failure? This question is particularly important for instructional programs that include new technologies, since the technology itself may pose unique barriers to implementation in a school setting (e.g., Newman, 1989; Robyn et al., 1989). Thus, our evaluation was sensitive to teachers' and students' attitudes about and use of the computer-aided aspects of the curriculum as well as their overall evaluation of the instructional activities.

Although the question of comparability of student outcomes between Math A and regular algebra students was of secondary importance, our design included regular algebra students as a "comparison" group.[1] We were interested in how the Math A students compared with "regular" algebra students on several dimensions, such as arithmetic achievement, algebra knowledge, demographics, attitudes, and other background characteristics. Knowledge of these types of differences might enable us to better understand effects (or lack of them) from the novel curriculum. Several of our attitude questions assess attitudes and beliefs about mathematics that reflect student experiences with traditional mathematics instruction; for example, that mathematics is mostly memorizing. If Math A students' attitudes show greater changes relative to algebra students in beliefs of this type, then we have some indication that the novel curriculum may have helped effect such changes.

STUDENT CHARACTERISTICS AND ATTITUDES

All 110 students in the four Math A classes consented to participate in the study. In the comparison group, 105 algebra students participated and three declined. Table 1 shows how these students compare with respect to age, grade level, gender, and pre-course arithmetic ability.

Math A students are older ($F = 4.86$, $p < .03$) and, on average, are in higher grade levels than algebra students ($F = 4.65$, $p < .03$). They also have significantly weaker arithmetic skills than algebra students ($F = 71.52$, $p < .001$). Algebra classes had a higher proportion of male students than the Math A classes. About 64 percent of the algebra students, compared to about 41 percent of the Math A students, expected a grade of B or

Table 1

Student Characteristics

	Mean Age	Mean Grade	Percent Male	Arithmetic Ability
Math A	15.1	9.8	52%	5.0
Algebra 1	14.8	9.6	57%	7.5

Total possible = 10.

---

[1] We use the term "comparison" group rather than "control" group because we are not using the data to examine equivalence of outcomes in the strict sense. We also acknowledge several differences between these groups of students that temper any comparisons made. Students were not randomly assigned to classes, and Math A was targeted at less-successful students who would typically be low achievers in a regular algebra class. According to the teachers' reports, confusion in the counseling office resulted in students of very low ability (including "learning disabled" students) being scheduled for Math A along with a few students who had already had algebra or geometry. Thus, Math A and algebra students constituted different student populations. In addition, the classes had very different instructional goals and curricula, although both courses covered some of the same material (e.g., solving linear equations).

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higher in the course. Algebra students liked algebra more than Math A students (71.4 and 51.8 percent, respectively; chi square 16.75, $p < .001$). Twenty percent of the algebra students judged algebra to be "easy," compared to only 4 percent of the Math A students (chi square = 10.75, $p < .005$).

We asked students several questions about their feelings toward mathematics and toward computers. Students answered on a four-point scale (1 = strongly disagree to 4 = strongly agree). Table 2 shows percentage agreement (percentage responding "3" or "4") with these statements for Math A and algebra students. Table 2 indicates that algebra students may have more confidence in their mathematics ability, although the difference between Math A and algebra students is not significant (item 1). Similarly, both groups look for external verification of their work (item 3), but the Math A students appear more disposed to do so. A majority of students in both groups think computers can improve algebra instruction (items 2 and 8). Items 4 and 10 assess how highly students value mathematics. Responses here appear somewhat inconsistent. About a quarter of the students in both groups saw little value in mathematics (item 4), but over half thought algebra was useful in life (item 10). Algebra students saw significantly more value in learning algebra than did Math A students. A second attitude about mathematics learning concerns discovery versus memorization (item 11). We would like students to see mathematics as a logical structure rather than as just a body of disconnected "facts." Most students believe that mathematics is mostly memorizing. In spite of this, a high percentage of students *also* believe that discovery is possible (item 6). Items 5 and 9 concern similar attitudes about mathematical thinking. Clearly, a majority of students believe that trial and error is a useful strategy. Few students, on the other hand, had faith in their ability to figure out a problem. This is perhaps consistent with their view that mathematics is mostly memorizing. If students believe that mathematics consists of memorizing formulas, then they will not go back to "first principles" to reason the answer to a question. Finally, item 12 concerns students' attitudes toward mathematics as a body of knowledge. Math A students appear significantly more likely to view mathematics as comprising unrelated topics than do algebra students.

We also asked students about their experiences with computers (see Table 3). Overall, students have a wide variety of experience with computers. Algebra students are significantly more likely to use computers outside of school (chi square = 13.08, $p < .001$) and to use computers to do their homework (chi square = 3.88, $p < .05$). A large proportion of students have taken courses about computers and nearly 40 percent of students in both groups have written computer programs.

Table 2

Attitudes Towards Math at Pretest

Item	Percent Agreement	
	Algebra	Math A
1. I am good at math	72.4	64.2
2. Many algebra courses could be improved with computers	72.4	69.4
3. In math, to know if you have the right answer, you must check with the teacher, a computer, or the book	53.3	64.2
4. I can get along well in everyday life without mathematics	22.9	23.1
5. A problem is easy to figure out even if I've forgotten exactly how to do it	28.6	25.7
6. In mathematics you can be creative and discover things for yourself	60.0	64.1
7. There is one right way to solve most algebra problems	36.2	33.3
8. Using computers to teach algebra is a bad idea	17.1	20.0
9. Trial and error can often be used to solve an algebra problem	58.2	68.0
10. It's important to learn algebra because algebra knowledge is useful in everyday life	67.3	54.1*
11. Learning mathematics is mostly memorizing	69.5	68.8
12. Mathematics is made up of unrelated topics	23.3	36.8**

NOTE: Because of missing data, Ns range from 103–105 for algebra students and from 96–110 for Math A students. (* = chi square 3.87, $p < .05$; ** = chi square 4.52, $p < .03$.)

In summary, the Math A students differ somewhat from traditional algebra students. They are older, in higher grades, and less skilled in arithmetic than algebra students. Math A students also like algebra less and are less likely to judge it "easy." Students in both groups have similar attitudes about mathematics, but algebra students see more relevance for algebra knowledge in everyday life. Algebra students have more experience with computers outside of school than Math A students.

Table 3

Computer Experience

Have You Ever...	Percent Responding "Yes" Algebra (N = 105)	Percent Responding "Yes" Math A (N = 110)
Taken a course about computers or computer programming?	70	65
Used computers outside of school other than for playing games?	75	51
Used computers in other classes?	50	49
Used a computer to do your homework?	36	24
Written a computer program?	42	39

STUDENT LEARNING

To assess changes in student learning, we administered an algebra achievement test to the Math A students before and after the curriculum unit, and, for purposes of comparison, to the algebra students after their instruction in solving simple linear equations.[2] Students also took an arithmetic test to determine initial arithmetic ability, apart from algebra knowledge. Finally, Math A students took two "unit" tests that measured learning of concepts specific to the new curriculum. Unit test results will be discussed in detail below.

Table 4 shows test scores for the 65 Math A students and 64 algebra students who completed all of their respective tests. This comparison eliminates students who entered the courses late, dropped out of the courses, or were absent on testing days.[3]

Algebra students scored significantly higher than Math A students on the arithmetic test ($F = 33.37$, $p < .001$) and algebra posttest ($F = 24.09$, $p < .001$). What is most striking,

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[2] We gave two tests to the Algebra I classes as well as the Math A classes. However, the algebra students cover the material on linear equations quite early in the semester, so the first of the tests was in effect a posttest, occurring immediately after students had received instruction in the relevant topics. The results of the second test, which was given at the end of the semester, are quite similar to the first and show a slight, statistically insignificant decrease toward the end of the semester (change score = -.2 on a test with 12 possible points). Thus, for purposes of comparing achievement, we use the first test given to the algebra students and the second given to the Math A students. On the other hand, where changes in attitude are concerned, we felt justified in using *both* algebra students' tests, since the issues in question relate to the way in which algebra is taught generally and is not content-specific.

[3] We conducted separate analyses to determine if these scores were significantly different for the group of 44 algebra and 49 Math A students who missed one or more of these tests. T-tests between total scores of students taking all tests as compared with those missing, for each of the three tests, were not significant. Thus, there is no bias with respect to achievement between these two groups.

Table 4

Arithmetic and Algebra Achievement Scores for All Students
(Means and Standard Deviations)

	Arithmetic		Pretest		Posttest		Change Score	
Students	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**
Math A	5.17	2.38	1.80	1.72	4.75	2.86	2.94	2.18
Algebra	7.42	2.04			7.07	2.50		

NOTE: Maximum possible scores are 10 on the arithmetic test and 12 on the pretests or posttests. Change score is the posttest minus the pretest.

however, is that both algebra students and Math A students did so poorly on the posttest, correctly answering only about half of the questions on average. Nevertheless, we were encouraged to see that the Math A students did show significant improvement from pretest to posttest ($t = 10.86$, $p < .001$). Moreover, among the Math A classes, Teacher A's students did significantly better than Teacher B's (change score 3.65 and 2.17, respectively; $t = 2.88$, $p < .01$).

We also examined group differences on two subsets of items: Six "innovative" items and six "traditional" items. Innovative items, for example, assessed students' ability to select (from a list of operations) the operation that would transform the equation to the next appropriate step or to identify the variables in a word problem. Traditional items required students to solve an equation presented in symbolic form (e.g., $3(Y - 2) = 12$; $5.2T - 10.3 = 10.5$). Algebra students completed about three (of six) traditional items correctly on the posttest. Math A students, on the other hand, do slightly better than algebra students on the innovative items at posttest, and show small gains from pretest to posttest.

For Math A students, the small gain in innovative items is encouraging, since the novel curriculum stressed these skills over solving symbolic equations. Analysis of subset test scores by teacher indicates that Teacher A's students had significantly higher change scores than did Teacher B's on both types of items.

Given such low test scores, these results are not very compelling from a curriculum developer's perspective. However, we have other data on the Math A students—teacher evaluations, unit tests, class notes, tutor usage—to determine which aspects of the curriculum were more successful than others and what kinds of changes might be made. We examine these data in more detail below.

MATH A STUDENT OUTCOMES AND CHANGES

Since the primary purpose of this evaluation is to assess how the novel curriculum changed students' knowledge, skills, and attitudes, below we analyze data for the Math A students alone. First, we more closely examine their achievement on the algebra test and unit tests. Then we assess changes in student attitudes. For these analyses, we are also interested in whether any teacher differences arise.

Math A Achievement

Although student performance on the algebra achievement test was low overall, some classes did better than others. Table 5 shows algebra achievement data by teacher. Both teachers' students had had equivalent arithmetic and algebra knowledge at pretest. Teacher A's students did significantly better at posttest than Teacher B's students (t = 1.99, $p < .05$). Change scores (posttest minus pretest) for the total test and various subsets of item types indicate that Teacher A's students outperformed Teacher B's students across the board.

Unit Test Results

Two unit tests were administered during the course. The first test was given after students completed curriculum topics on identifying variables, developing qualitative models, and exploring covariation using tables and graphs. The second unit test was given after completion of the full curriculum module and assessed the construction of quantitative models (equations), identification of target variables, equation solving (at goals, operations, and symbol manipulation levels), problem checking, and interpretation of answers. Appendix B contains both unit evaluations. Both unit evaluations were paper-and-pencil tests that mirrored noncomputer and computer assignments.

Table 5

Algebra Achievement by Teacher

	Teacher A		Teacher B		Significance
	Mean	S.D.	Mean	S.D.	Level
Arithmetic	4.82	2.14	5.25	2.60	n.s.
Pretest	1.63	1.47	1.72	1.98	n.s.
Posttest	5.33	3.10	4.15	2.75	p <.05
Total change	3.65	2.26	2.13	2.29	p <.01
Equation-solving change	1.76	1.65	0.92	1.66	P <.02
Innovative change	1.89	1.30	1.21	1.32	p <.02

NOTE: Maximum possible scores are 10 on the arithmetic test, 12 on the pretest and posttest. Change scores represent average change from a base of six items each.

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Table 6 shows average student scores, by teacher, on each of the two tests. In contrast to their performance on the algebra achievement test, both teachers' students did fairly well on the first unit tests, although less well on the second. Teacher A's students scored slightly higher than Teacher B's students, but this difference is not statistically significant.

To examine student achievement in greater detail, we constructed several subsets of items and, for the sake of comparison, standardized subset scores on a ten-point scale (see Table 7). These data reveal several important differences. First, students were twice as good at solving symbolic equations that were situated in a real-world problem than solving similar equations presented abstractly. In a "situated" equation (see Fig. 3 for an example), the student is asked to solve a symbolic equation derived from the word problem and word graph. The student is also asked to identify the target variable and the goal *before* solving the problem. Students did better on these types of problems than on similar equations (e.g., $3Y + 9 = 12$) presented alone. This finding reinforces the belief that problem-solving should be grounded in meaningful contexts so that students can learn to "mathemetize" or construct the links between formal algebraic expressions and the actual situations to which they refer (e.g., Clement, 1982). It appears that meaningful problems, such as those included in our

Table 6

Unit Test Results by Teacher

	Teacher A			Teacher B		
	N	Mean	S.D.	N	Mean	S.D.
Unit Test #1	49	5.01	1.39	58	4.80	1.46
Unit Test #2	55	4.20	2.23	53	3.72	2.61

NOTE: Maximum score for #1 and #2 were 6 and 11 points, respectively.

Table 7

Standardized Subtest Scores by Teacher

	Teacher A		Teacher B		
Subtest	Mean	S.D.	Mean	S.D.	
Symbolic equations	2.27	3.05	2.59	3.46	
Situated equations	4.80	3.14	4.11	3.56	
Find target	7.63	3.71	5.75	3.97	$p < .01$
Choose goal	5.89	3.18	4.60	3.48	$p < .05$
Choose operator	3.45	3.95	2.36	3.74	
Check answer	1.64	2.73	1.51	2.99	
Interpret answer	3.67	3.98	3.58	3.62	

NOTE: Maximum score on each subset is 10.

curriculum, can improve over current mathematics instruction, which does not engage students' interpretive and meaning construction capacities (cf. Resnick, 1987). The challenge, then, is to determine what constitutes a "meaningful" problem to teenage mathematics students. We obtained some feedback from students on this and other aspects of the curriculum, which we report below.

Table 7 also indicates that students did fairly well at identifying the target variable and choosing the goal for a particular step. Furthermore, Teacher A's students did significantly better than Teacher B's on these two types of items. Our classroom observation data indicate that Teacher B did not reinforce target identification in her presentation, whereas Teacher A covered this topic in at least two different sessions and received an average rating of 2.5.[4]

Students did less well at checking and interpreting answers. The low score for "checking" is somewhat disappointing. There is some evidence that successful math learners engage in more metacognitive behaviors, such as checking. It seems reasonable that a less-routinized approach to mathematics, such as that fostered in our curriculum, could substantially improve learning, including developing metacognitive or strategic behaviors for solving problems (cf. Resnick, 1987). However, it appears that students failed to check even when they were specifically instructed to do so. It may be that checking is not a typical activity for these students and that the instruction alone did not induce them to pick up this metacognitive habit. Our observational data indicate that neither teacher reinforced checking during their classroom instruction: Only one observer noted any presentation of this topic, to which a rating of "1" (not effective) was assigned.

Changes in Attitudes

Next we examine changes in attitudes toward mathematics. Here, we are interested in whether Math A students' attitudes change in the anticipated direction (either an increase or decrease in agreement, depending on the statement) relative to algebra students. This analysis compared what percentage of Math A students' (N = 65) attitudes changed relative to the algebra students (N = 64).

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[4] Observers in each class noted topic numbers from a list of possible topics and rated the teacher's presentation on a three-point scale: 1 = not effective, 2 = satisfactory, and 3 = very effective. Observers were members of the research team who had helped develop the curriculum and thus were knowledgeable about curriculum objectives and intent. Although their ratings may be very subjective, they do give some indication of whether the teacher covered the curriculum material or taught it as originally intended.

Students in both groups tended to have higher expectations for their course grade at posttest. We observed a dramatic shift, however, in their feelings about algebra. As reported above, at pretest the algebra students liked algebra significantly more than did Math A students. By the end of the course, Math A students' feelings changed in the positive direction, and algebra students' feelings were more negative: 42.2 percent of Math A students had more positive feelings, compared with only 9.4 percent of algebra students (chi square = 22.65, $p < .001$). Over a third of the algebra students reported more negative feelings about the subject.

Students' feelings about the difficulty of algebra also changed. Although the difference was not significant (chi square = 9.10, $p < .06$). Math A students felt they had less difficulty with algebra (21.5 percent changed in the direction of "easier") whereas algebra students had more difficulty (21.9 percent changed in the direction of "harder").

We also found several important changes in attitudes toward mathematics (see Table 8).[5] First, algebra students were less likely than Math A students to change their attitudes in either direction. As Table 8 indicates, Math A students were more likely, at the end of the course, to believe that correct answers must be checked with an outside source. This may be due to their experience with the computer during the semester, since the computer could check the correctness of any solution line or problem at the student's request. Math A students increased their belief in their ability to figure out a problem, whereas algebra students felt less able to do this. Math A students were also more likely to discount a trial-and-error approach to problem solving. Over a third of the Math A students increased their perception of algebra as useful for everyday life; one-fourth of the algebra students showed a decrease in this perception. This suggests that we were somewhat successful in our attempts to show that algebra is a useful tool for everyday life. It also suggests that learning algebra problem solving in the context of real-world situations (as did the Math A students) may demonstrate this usefulness to students, as opposed to learning algebra in the context of abstract symbol manipulation. Finally, Math A students were less likely to see mathematics as comprising unrelated topics. This suggests that the curriculum achieved its intended goal of teaching students to see the relationships between mathematical concepts and different representations of the same concept.

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[5] To measure attitude changes from pretest to posttest, we calculated a difference score by subtracting the pretest rating from the posttest rating. A positive result indicated more agreement with the statement, a negative result indicated less agreement and a zero result indicated no change in the student's attitude. Chi-square tests (student type by change) determined significant changes between groups.

Table 8

Significant Changes in Mathematics Attitudes

	% of Students at End of Course Who			
	Disagree More		Agree More	
In math, to know if you have the right answer, you must check with the teacher, a computer, or the book	A	14.6	A	25.0
	M	24.6	M	41.5
A problem is easy to figure out even if I've forgotten exactly how to do it	A	23.4	A	7.8
	M	18.5	M	29.2
Trial and error can often be used to solve an algebra problem	A	10.9	A	28.1
	M	30.8	M	20.0
It's important to learn algebra because algebra knowledge is useful in everyday life	A	25.0	A	12.0
	M	16.9	M	35.4
Mathematics is made up of unrelated topics	A	15.6	A	14.0
	M	32.3	M	21.5

NOTE: A = algebra students, M = Math A students. For all items, $p < .05$.

CLASSROOM OBSERVATIONS

At least one observer was present in every class and completed an observation form (described in Sec. 3). These observations provide data on time allocated to various classroom activities, assessments of teacher effectiveness for the various topics, and general assessments, such as how well the students did with the computer.

On average, teachers spent about 12 minutes at the beginning of the class to "settle" the students and warm up for the lesson. This was followed by about 14 minutes of lecture and 28 minutes for either noncomputer or computer work. The two teachers varied somewhat in the time spent on these activities. Teacher A spent more time for warm-up and lecturing (about 29 minutes) than did Teacher B (about 22 minutes; $t = 2.64$, $p < .01$). As a result, Teacher B's students had about 10 minutes more computer or noncomputer time than Teacher A's students ($t = -3.60$, $p < .001$).

Observation data show that Teacher A circulated among the students working at the computer more than Teacher B (about 87 percent and 17 percent of classes, respectively; chi square = 9.76, $p < .002$). As mentioned above, this matches their respective interest in working with the computer. Observers also gave general assessments of how well the students did with the lesson and how well they worked at the computer. Teacher A's classes received higher ratings than Teacher B's in both areas. Observers judged students performance with the lesson as "satisfactory" or "very well" about 86 percent of the time for Teacher A, as compared to 48 percent for Teacher B (chi square = 7.013, $p < .01$).

Performance with the computers was "satisfactory" or "very well" for 89 percent of Teacher A's classes and 50 percent of Teacher B's classes (chi square = 7.127, $p < .01$). Observers also noted that computer crashes disrupted student work somewhat in both classes—"a lot" in 21 percent of Teacher A's classes (4 of 19 classes observed) and 29 percent of Teacher B's (5 of 17 classes observed). Overall, computers were problem free only about one-fourth of the time. On average, students in Teacher A's classes spent about 2.40 hours using the computer, and Teacher B's students spent about 2.22 hours ($F = 0.65$, $p < .4215$). However, the distribution of time over students was different, with a median time of 2–3 hours for Teacher A and 1–2 hours for Teacher B (chi square = 18.86, $p < .009$). Since computer crashes interrupted the time-keeping program, these numbers underestimate the actual time spent.

STUDENT EVALUATION OF THE CURRICULUM

As part of the posttest, Math A students were asked to evaluate the curriculum. Student satisfaction with various aspects of the curriculum is shown in Table 9. The first seven items in Table 9 register overall impressions about the course. These data indicate that about half of the students enjoyed the unit on "Modeling Real World Situations." They found the situations (e.g., car wash, Heal the Bay) more realistic than interesting. Teacher B's students found the situations more interesting than Teacher A's (mean rating 2.53 and 2.2, respectively, $F = 5.16$, $p < .03$). Three-fourths of the students enjoyed using the computer and 70 percent found the computer hints helpful. The computer was easy to learn to use and most students would use it again.

Item 8 asked students to rate how helpful working with a partner, computer assignments, or noncomputer assignments were for doing the work on the modeling unit. Responses were very consistent across both classes and show that working with a partner was the most helpful. Furthermore, computer assignments were more helpful than noncomputer assignments.

Item 9 asked students to rate, using the same scale described above, how helpful various exercises were for understanding situations mathematically. Overall, most students felt these different exercises were helpful. However, students varied in their responses across teachers. Teacher A's students found making a table, using a graph, and writing a word equation more helpful, whereas Teacher B's students preferred making word graphs and writing symbolic equations. Finally, students rated helpfulness of various activities for learning to solve an equation (item 10). Again, student ratings of helpfulness were high overall and fairly consistent across teachers.

Table 9

Student Evaluation of the Curriculum

Item	Percent Reporting Positively		
	Teacher A (N = 40)	Teacher B (N = 49)	Total
1. Enjoyed modeling unit	55.0	51.0	52.8
2. Situations interesting	32.5	49.0	41.6
3. Situations realistic	47.5	55.1	51.7
4. Enjoyed using computer	77.5	67.3	71.9
5. Computer hints helped	50.0	67.3	59.6
6. Would use computer again	87.5	98.0	93.3
7. Computer easy to learn	92.5	89.7	91.0
8. Help doing work for unit:			
Partner helped	73.7	70.8	70.9
Computer helped	60.5	60.4	60.5
Noncomputer assigments helped	52.6	52.1	52.3
9. Help learning to understand math situation:			
Word graph helped	55.3	60.4	58.8
Table helped	68.4	64.6	66.3
Graph helped	73.0	66.0	69.0
Word equation helped	76.3	53.2	63.5
Symbolic equation helped	39.5	55.3	57.6
10. Help learning to solve equation:			
Picking target variable helped	65.8	62.5	63.9
Naming goal helped	65.8	76.0	71.4
Naming the operation helped	65.8	63.8	64.7
Doing the math helped	73.7	74.4	74.1

NOTE: Percentage responding "3" or "4" on a four-point scale, with 1 = no help, 2 = somewhat helpful, 3 = helpful, 4 = very helpful.

We also asked students to indicate, from a list of choices, what or who was helpful in helping them learn to use the computer. Percentage of students choosing each item, by teacher, are shown in Table 10. Overall, students felt that a member of the research team was most helpful for learning to use the computer, followed by just trying it out themselves. About a third of the students found help from their partners. Class instruction and teacher help were more beneficial in Teacher A's class than in Teacher B's (for "teacher helped learn," F = 11.61, p < .001). This result is not surprising since Teacher A was more interested in learning about the computer and spent extra time getting familiar with the software. Teacher B, however, decided not to involve herself with the computer and looked to the researchers to handle any questions or problems with the machines during class time. Teacher A's greater knowledge made her classroom presentation more comprehensive and effective and made her a better resource when a student had difficulties. Thus, Teacher A's students found less need for the worksheet that explained computer operation than did Teacher B's.

Table 10

Student Evaluation of Help to Learn Computer Use

	Percent Choosing Item		
	Teacher A (N = 40)	Teacher B (N = 49)	All
Class instruction	57.5	40.8	48.3
Worksheet	15.5	24.5	20.2
Partner	35.0	34.7	34.8
Teacher	65.0	30.6	46.1
Researcher	60.0	59.2	56.2
Trying it out myself	57.5	53.1	55.1
Other	15.0	8.2	11.2

Finally, we asked students what they thought were the "best" and "worst" features of the unit on modeling real-world situations. We coded students' open-ended responses into the categories shown in Table 11. In both classes, the computer was most often named as the best feature of the unit. Several students in both classes named the "situations" as both the best and worst feature of the unit. Students seemed to dislike the equation-solving aspects of the curriculum, particularly those in Teacher A's class. Some students also had problems with the computer interface or found the curriculum too boring. Eighteen percent of Teacher B's students, compared to only 3 percent of Teacher A's, found the unit too difficult.

Table 11

Best and Worst Features of the Curriculum

	Percent Reporting			
	Best		Worst	
Item	Teacher A (N = 33)	Teacher B (N = 40)	Teacher A (N = 36)	Teacher B (N = 43)
Situations	12.12	25.00	13.89	18.60
Modeling	6.05	7.50	—	—
Computers	39.39	27.50	5.13	16.28
Computer feedback	6.06	2.50	—	—
Friend/staff	6.06	2.50	—	—
Noncomputer work	3.03	5.00	—	—
Solving	0.00	5.00	—	—
Everything	15.15	7.50	2.78	4.65
Nothing	6.06	7.50	13.89	16.28
Interface	—	—	16.67	4.65
Equations	—	—	25.00	11.63
Too boring	—	—	5.13	4.65
Too hard	—	—	2.78	18.60
Other	6.06	10.00	11.11	4.65

5. CONCLUSIONS

This study had two broad goals: testing a novel approach to algebra that focused on helping students learn skills for building and using mathematical models to solve real everyday problems, and developing techniques for the successful transfer of technology into the classroom. This pilot project gave us useful insights into both the effectiveness of the curriculum itself and of the implementation process for a computer-integrated course. In this section we discuss our findings concerning the techniques for technology transfer and the effectiveness of the curricular approach, and we conclude by proposing curriculum modifications based on our findings.

TECHNOLOGY TRANSFER

Overall, as described in the evaluation section, the technology transfer was effective, insofar as the computers functioned as a standard component of daily lesson activities, typically used by students twice a week. Thus the technology transfer model, whereby a team of researchers and teachers provide computer-integrated lesson plans and course materials, proved successful in enabling substantial computer use for new activities. The integration of computer tasks directly into lesson materials is an implementation approach that differs radically from the more typical practice in which a teacher occasionally takes a math class to a lab to use a software package that seems to fit some aspect of a lesson, for example, providing drill and practice in an activity already available in the textbook. In implementing the technology transfer, we were also concerned about methods for providing student and teacher training and making effective use of a limited number of machines.

Teaching students to use the computers and their various environments was accomplished as a whole class activity requiring only a moderate amount of class time. In classes where students received the lessons designed to train them in using the computer (Teacher A's classes), students easily completed assignments with a minimum of staff or teacher coaching. Moreover, students reported enjoying using the computers and were able to master the operation of the various environments.

Class observation records and student reports indicate that the cooperative work groups for computer and noncomputer assignments were effective in sparking discussion and reflection. Given economic and logistical considerations, classrooms are likely to have only a limited number of computers (Office of Technology Assessment, 1988) and thus sharing becomes a natural tactic to increase student computer time. However, we found that this necessity proved to be a surprising asset, if organized properly. We found that having specific tasks for each student helped keep one student from dominating the activity. For example, we had one student do the mouse clicking while the other input material from the keyboard, and this process kept both students active while inspiring some thoughtful negotiation about which data were to be input. The necessity of sharing produced effective work sessions. It would be valuable to have a more controlled study of the benefits of cooperative versus independent learning and to extend the work of Leinhardt (1989) in determining which tasks are best accomplished in a group and how group work can be managed for the most effective productivity.

Teacher comfort with the technology varied greatly as was reflected by the variation between Teacher A and Teacher B in the time spent instructing students in computer operation and circulating among students working at the computers. Increased training that includes sufficient time for teachers to become familiar with computer capabilities and assignments and with the operation of the machines is necessary to help teachers become more comfortable with the new teaching tool.

CURRICULUM EFFECTIVENESS

The novel algebra curriculum was intended to teach students to acquire several skills, including identifying the variables in a real-world situation; describing the covariation of the values of dependent variables; recognizing and interpreting multiple representations of information (tabular, graphical, and equational); constructing qualitative and quantitative models of situations; solving problems on several levels (specifying solution goals and operations and performing symbol manipulations); and interpreting solutions in terms of the original problem.

The curriculum achieved mixed effectiveness. In general, students were more successful at creating and using qualitative models than quantitative models. In the qualitative modeling section of the curriculum, over two-thirds of the students could successfully identify the variables, build qualitative models, gather data to explore relationships, interpret tabular and graphical representations, and describe relationships. In fact, they outperformed the algebra students on these innovative items—a significant result given that the Math A students' basic math abilities were significantly less than their peers in the algebra classes.

Students were less successful at constructing quantitative models and equation solving. The class unit tests indicate that only about a quarter of the students could specify the three operators necessary to transform a word graph to a word equation; about half of the students could specify the appropriate goals; about a third could specify the appropriate operation; 45 percent could perform the symbol manipulations to solve problems arising from real-world situations; and 24 percent could perform the symbol manipulations to solve abstract problems. However, attitudes showed considerable improvement. In our past field tests of the algebra tutor, using abstract problem solving, we frequently were asked, "When (in our lives) are we ever going to use this stuff?" Students' improvement in attitude toward the utility of algebra seems to indicate the effectiveness of using a real-world context for problem solving to overcome this objection.

Four factors appear to have influenced our results: (1) the target population we selected, (2) teacher comfort with the course goals and new roles, (3) computer operation, and (4) curriculum considerations. First, our results are complicated by the limited mathematics skills of the target students. Though final outcomes were low, we were encouraged by the substantial gains in skills made by students during the study.

The second factor that may help explain our results is the difficulty the teachers had in adopting a new approach and new roles. Early in the course, Teacher A, who had worked closely with the development of the curriculum materials and the software, was intellectually committed to the course goals, and in class observations was judged more effective in presenting the curriculum, made a revealing comment. She remarked that it was extremely difficult to teach the material because "the lesson is not mine—the vocabulary is not mine." She hoped that after a few more days it would feel more natural. The teacher's comment points up the difficulty of assimilating new approaches and new theoretical goals even after extensive involvement in the course development.

Our class observations often noted the teachers' difficulties in this area. For example, in teaching about problem-solving goals, the teachers tended to use the goals (e.g., ISOLATE, DISTRIBUTE) as process labels rather than as the strategic steps that would lead to the determination of the value of the unknown variable. The emphasis was typically on what to do, not why to do it. The teacher would say, for example, "when you have a term added to the variable, the instruction you give to the computer is ISOLATE." The teachers also had difficulty relating the equation solving to real-world situations, and they tended to gravitate to the more familiar symbol-manipulation techniques. The framework for solving the problem was thus often lost and with it the reason for solving for the unknown variable.

Not only were the teachers required to use a new approach but they also had new roles introduced by the use of the computers. Both teachers were familiar and comfortable with managing small groups working simultaneously on assignments and did not have difficulty with the logistics of managing computer and noncomputer assignments. But Teacher B had difficulty interacting with students working at the computer and essentially abdicated any teacher role in computer work for most of the course. She did not feel comfortable with the software and could not allocate the time necessary to learn about its capacities or about problems students might encounter using the computers. Her introductions to the computer environments were cursory and she relied instead on project staff to introduce students to the computers and to coach their work. Teacher A circulated easily among students doing computer and noncomputer work. However, she was sometimes frustrated by her lack of expertise in dealing with computer problems, which she felt diminished her control and authority in the classroom, since she was dependent on project staff to conclude assignments. New approaches to mathematics and new roles introduced by advanced technology require teacher training to enable teachers to assimilate new goals and become comfortable with course material and tools.

A third factor contributing to our results was the slowness of the computers running the software and the frequent bugs causing computer crashes. Though students frequently had 25–30 minutes available to work at the computers, their actual time was restricted by machine difficulties. Thus the computers did not entirely assume the teaching role they were expected to have, placing more of a burden on the teachers and limiting student learning while the students waited for computer access. To be effective in a classroom situation, software needs to be error free and hardware processing must occur with reasonable speed.

Finally, to understand our results, we looked more closely at the equation-solving activities in the curriculum, which were the activities that students found most difficult. Though their overall posttest equation-solving scores were low, students did make significant gains in their skills, particularly in nonabstract equation solving. Students were twice as successful in solving equations that arose from real-world situations as they were in solving exactly the same type of equation presented abstractly. We speculate that situating equations allows students to draw on their intuitive and experiential resources and is more intrinsically motivating. This conclusion supports contentions that curriculum that expands students' understanding of the mathematical representations of real-world situations is both more engaging for students and trains students in skills demanded in the modern workplace (see Romberg, 1987; and Office of Technology Assessment, 1988).

A secondary problem with the equation-solving portion of the curriculum was terminology, which became a burden for teachers and students. In particular, rather than reifying or making explicit problem-solving actions, the labels for equation-solving goals seemed abstract to students (and teachers) and became just one more meaningless bit of information to be memorized. Additionally, the pacing of assignments meant that the second half of the course, on equation solving, was handled much more quickly than the modeling section. Students had very little opportunity to practice direct symbol-manipulation techniques. Only one computer assignment asked students to directly solve equations. We had expected, however, that the familiarity with goals and operations would produce an easy transition to direct symbol manipulation. Students' poor equation-solving skills may indicate that more practice in direct symbol manipulation is desirable or that the top-down goals approach to problem solving was not effective in promoting acquisition of equation-solving skills. Students may require more familiarity with symbol manipulation before they are able to specify problem-solving goals.

CURRICULUM MODIFICATIONS

In addition to test results and student feedback, our classroom observations and teacher feedback suggest several needed changes to the curriculum and supporting tutor software.

1. The connection between the qualitative and quantitative components of the curriculum need to be made clearer. Students did not always relate the graphical and tabular models to equational ones. A clearer connection needs to be established between the word equation students constructed in preliminary modeling and the symbolic equations they are later asked to solve. Graphical and tabular activities should be integrated into the equation-solving section of the curriculum.
2. New graphical representations need to be designed and integrated with existing quantitative representations. For example, tutor graphing capacities need to be expanded so that students can plot their own points as they collect data and gain a better understanding of the equation line.
3. Students need more coaching on the construction of equations to represent natural situations, particularly selecting appropriate operators. A computer activity letting students explore the relationship between two codependent variables would be one helpful approach.
4. Students are reluctant to read any textual instructions. Having students read aloud to staff indicated that they were comfortable with the vocabulary, but there seems to be a universal principle operating of doing the least possible work. More noticeable messaging techniques (e.g., computer-generated voice messages) need to be investigated.
5. "Mindlessness" (Salomon, 1985) is a related problem. To accomplish a task, students will often plug in values, or click the mouse through a menu of options with no plan in mind, until the computer performs the task. Motivation and engagement thus become

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key concerns. On a technical level, this argues for the elimination of menus, or perhaps a staged introduction of menus once students have mastered doing activities "by hand." In addition, it will be important to increase the perceived cost of making a high number of errors—a frequent effect of mindless choices. In constructing computer activities, we also need to guard against contrived activities and to develop activities that require thinking and engagement. For example, the sequence of activities to develop and transform a qualitative representation from a word graph to a symbolic equation was tedious and lent itself to the mindless use of menus.

6. Students need more instruction in inquiry techniques and systematic ways to collect data (e.g., holding all but the one variable under investigation constant when studying covariation) and in reflective techniques (e.g., looking at the effect of a problem-solving command on the complexity of the resultant equation). Attention to the way assignments are framed should also help guide students in more thoughtful behavior.
7. Course terminology needs to be revised so that it is more meaningful. Learning and teaching terminology became a main stumbling block of the course, focusing attention on the names for mathematical processes rather than the reasons for them.
8. More attention needs to be given to providing students with direct practice in symbol-manipulation skills. It may be advantageous to introduce some practice with symbol-manipulation skills together with the use of explicit goals and operations.
9. We must continue to develop interesting and realistic natural situations for students to explore.

NEXT STEPS

Our next step is to develop an expanded version of the curriculum module described above, which will include material on statistics and functions.[1] We will use the technology transfer model successfully fielded in this pilot but will devote more attention to teacher training and teacher involvement in the initial definition of course goals. We also plan to upgrade the computer configuration and to pursue extensive lab pilots of the revised and new software. The existing curriculum module will be modified in accordance with the recommendations outlined above and the results of preliminary lab studies. We plan to test the expanded curriculum with both Math A and algebra students in the 1990–1991 school year.

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[1] The new curriculum development and test is funded by a grant from the U.S. Department of Education Fund for the Improvement and Reform of Schools and Teaching.

Appendix A

CURRICULUM MATERIALS: TEACHER LESSON PLANS AND STUDENT MATERIALS

TEACHER LESSON PLANS

ALGEBRA CURRICULUM: INTRODUCTION TO LINEAR EQUATIONS UNIT

MODELING REAL WORLD SITUATIONS

Theoretical concepts:

1. Equations are models of real-world situations. Situations embed various features or natural variables
2. Real world situations can be analyzed in terms of natural variables.
3. Values of dependent variables covary. Several kinds of questions about the qualitative relationship among dependent variables can be answered, and in some cases the specific values of a variable can be determined by using information derived from exploring dependency relationships.

Behavioral goals:

1. Students will be able to enumerate variables in natural situations
2. Students will be able to use the computer tools.
3. Students will be able to describe dependency relations through the use of word graphs and tables
4. Students will be able to classify various kinds of qualitative covariation relationships (e.g. increasing, decreasing, non- monotonic)
5. Students will be able to answer questions about natural situations (both discovery types of questions, such as finding the unknown, and achieving specific value types) using various intuitive means like guess-and-test and tables of values
6. Students will become familiar with the semantics of graphs and tables
7. Students will be able to extend a given graph/table by predicting or estimating further values
8. Students will be able to translate the variables in a given situation into word equations

Pedagogy:

1. The teaching style should be facilitative, eliciting information from students, and encouraging students to become increasingly independent in their work. Teachers should provide the scaffolding on which students can build their own understanding. The teacher's goal should be to fade-out her coaching as students become independently able

2. Review, introduction of new topic, guided practice, and independent practice are the typical lesson elements.

3. Understanding of the use of equations as models for real world situations should be developed before introducing equations; thus, the unit on linear equations begins by introducing students to the elements of equations: (i) variables as quantifiable descriptors of natural situations, (ii) the dependencies among variables, (iii) the way variables covary.

4. Students should focus on the concepts of variables and covariation before they are able to write or solve equations.

5. Multiple forms of representation (wordgraphs, tables, Cartesian graphs), are used so that students see that equations are only one way of getting information about a problem, and that data can be represented in various ways.

6. Students gain understanding by observing and recording changes, and answering questions that lead to generalized understanding.

7. Cooperative learning groups are used to enhance motivation, sharing of ideas, and course ownership

LESSON 1: INTRODUCTION, IDENTIFYING VARIABLES, WORD GRAPHS

Goals:

1. Introduce unit topic: equations are models of real-world situations
2. Introduce concept of "variables" (situations embed various natural features or variables).

Activities:

1. Warm-up activity. (Preserves class routine)
2. Teacher briefly introduces unit.
3. Teacher presents a topic, e.g. School Fundraising Dance
   - Students generate lists of variables for a given domain (brainstorm)

     E.g., Variables: # of students
     price of tickets
     entertainment (band, cassettes, sound system)
     food (soda, popcorn, candy)
     ecurity
     clean-up crew
     nice atmosphere (decorations)
     ate-time-place (non measurable items)
     t-shirts
     raffles
     dance contest prizes
   - Teacher refines list to quantifiable items, translating relevant items into measurable items (e.g. nice atmosphere=decorations)
   - Teacher, with student input on variables, writes word graphs to define "Profit", e.g.

     #students ticket price expenses > profit

     OR:

     #students ticket price (food #students) band
     security decorations > profit
4. Students generate variable list for another domain. Teacher records list on board and checks for appropriate items.
5. Teacher identifies output variable and asks students to write word graphs showing other variables that influence the output variable (input variables) and share with class. (Teacher models word graph depiction).
6. Teacher divides class into groups. These groups will remain stable for seat assignments for the duration of this unit. Allow about 20 minutes for the entire activity. Give students a warning at about 8 minutes so that they can finish and hang up their charts.

Activity directions:

1. Ask each group to select a recorder and presenter
2. Working on large sheets of newsprint, each group formulates a list of quantifiable variables for a different domain.
3. When the list is completed, the recorder should raise his/her hand, and the teacher will help the group identify an output variable.
4. The group should construct a word graph for the domain
5. When the chart is finished, the recorder should bring it to the front and hang it on the blackboard.

7. Teacher should review each list, checking for quantifiable items, and eliciting additional items from the class. Have a student from the group share the word-graph with the rest of the class.

VOCABULARY: Variable, word-graph, output variable, input variable

LESSON 2: INTRODUCTION TO THE COMPUTER

Goals:

1. Practice enumerating variables of natural situations
2. Investigate dependency relationships through the use of word-graphs and tables, and exploratory and problem oriented computer activities
3. Answer questions about natural situations using intuitive means such as guess-and-test, and tables of values
4. Become familiar with computer operation

Class Activities

1. Review variables and word graphs by having students brainstorm variable list together. After teacher identifies an output variable, students should suggest input variables to complete a word graph.

2. Teacher introduces computer through whole-class worksheet covering keyboard, mouse, login; and specifically the screen format and problem input for word-graphs (menu, value boxes, table of values). SEE LESSON SCRIPT AND VIEWGRAPH MASTERS ON FOLLOWING PAGES.

— Worksheet also allows teacher to walk students through a sample computer problem and then gives student a chance for guided practice. Using the worksheet, the teacher explains how the tables will be built, the type of questions students will encounter, and how to enter data.

3. Groups A & B working in pairs complete Computer Assignment #M-1. Groups C & D work on Seat Assignment #M-2. Sample worksheets follow this lesson.

Computer Assignment #M-1:

1. Select Topic. Students will be presented with a menu of topics. All students should select CAR WASH for their first topic. Worksheet M-1 is directly keyed to the CAR WASH topic. After completing CAR WASH, students may select any topic. They will not need to complete a worksheet for subsequent topics chosen for the first assignment.
2. Formulation. Students will select the appropriate variables for the topic word graph.
3. Unconstrained exploration. Students will begin by entering values into any of the word graph boxes. They will have an opportunity to vary the values at will. Their explorations will be recorded in a table. Students will have an unconstrained opportunity to explore covariation.
4. Constrained Exploration. In this activity, all but two of the variables will be anchored. Students can change values in one box of the word graph and observe how values in the other box covary. A table of values will record students' work. Students are given various kinds of qualitative questions to answer about the relationship between the unanchored variables.

Seat Assignment #M-2: Students will explore situations by answering questions showing their comprehension of the situation, identifying variables in the situation, constructing wordgraphs, filling in tables, and answering questions that involve reading tables.

VOCABULARY: word graph, computer-terms

INTRODUCTION LESSON: THE RAND COMPUTER TUTOR

1. **Introduce the computer:** Today, we are going to begin using the computers to do some exploring and problem solving. You'll be looking at situations and variables like we were doing in class yesterday. The computer will let you explore what happens when you enter different amounts for the variables. You'll be using tables (and later in the week, graphs) to help you keep track of what you find out. Everybody will work in pairs at the computer. I'll assign pairs after we have a look at how to use the computer. Be sure to work only with your assigned partner so that the computer can keep track of your assignments.

2. **Distribute Computer Get Acquainted sheets.**

3. **Review computer operation:**

A. LOGGING ON - SLIDE 1

1. **Login:** The computer will ask the student to type in their Student ID #. Both partners should type in their ID #'s. Students should check to see that they have typed their numbers correctly. Use DELETE or BACKSPACE to correct any errors, and then hit RETURN. If one partner is absent, only enter the ID of the student using the computer.

2. **Choose a topic:** Every time you finish logging-on, the computer will put up a menu and ask you to select a topic. The first time you use the computer, please select CAR WASH, after that choose any topic that interests you.

3. **The Mouse:** To select a topic, move the mouse on its board to move the arrow on the screen onto the topic, then click the mouse. You can click any button. One partner should plan to handle the mouse, and the other partner should plan to type. Trade jobs about half way through.

B. SCREEN - SLIDE 2

1. Point out the position on the screen of:

- The Situation
- The Word Graph
- The Instructions window
- The graph and table

2. Ask students to draw a circle around the Instructions window, or mark it with a colored pen. **STUDENTS SHOULD ALWAYS BEGIN BY READING THE INSTRUCTION WINDOW TO FIND OUT WHAT TO DO.**

C. KEYBOARD - SLIDE 3 Point out position of numbers; ask students to put a check on the RETURN key, and then to circle BACKSPACE and DELETE.

D. PICKING VARIABLES - SLIDE 4

1. **Instruction Window:** Ask a student to read the instructions in the Instructions window. Sometimes a special message will appear (ADD EXTRA MESSAGE TO SLIDE: "Click on the

OUTPUT variable. Click here to continue.") The special messages have a black bar. Click on the black bar to get rid of the message and continue on.

2. Situation Window: Ask a student to read the situation.

3. Menu:

- The first task is to select the OUTPUT variable. Explain OUTPUT (e.g. the result in the situation; the overall concern in the situation). Ask students to select the OUTPUT variable. Have students write the correct OUTPUT variable in the wordgraph on their sheets.
- Explain INPUT variables (e.g. the factors that the output variable depends on). Ask students to name the INPUT variables. Tell students that two on the list are wrong. If students select an incorrect variable explain why it is wrong (non-quantifiable, or insufficient information in the situation). Have students write the correct variable names on their sheets.

E. EXPLORING WITH DIFFERENT VALUES - SLIDE 5

1. Instruction Window: Ask a student to read the instructions.

2. Entering numbers: Explain that students will click on a variable box, the box turns black, the student types a number and then must hit RETURN, then the number appears in the box. To correct an error, click on the box again and enter the number you want.

Elicit values from the students that they think are realistic. Under each variable box, there is a guide to the appropriate units for the answer. Cents are entered as decimals, e.g. ".04". Have students write the numbers in the variable boxes. If necessary, help students arrive at realistic values by offering ranges, e.g. MILES will probably be between 300–1200; EXPENSES might be 2–6 cents/mile; INSURANCE might be $50 - 1500/mo.

Explain that as soon as students fill in all but one of the boxes, the computer will compute the last value.

Simulate the computer, compute and write in the output value. Have students write the value for the output variable.

REMEMBER: To enter numbers: CLICK, TYPE, hit RETURN

3. Table of values: After the student enters values for all of the input variables, the set of exploratory values will be automatically entered and saved on the table. Have students enter the numbers from the word graph onto the table.

4. Assignment sheet: After trying 4 sets of numbers, students are asked to fill out their assignment sheet. Partners should work together to complete the assignment. Each student should fill out a copy to keep.

F. EXPLORING WITH ONE FREE VARIABLE BOX - SLIDE 6

1. Instruction Window: Ask a student to read the instructions.

2. Typing the GIVENS: Explain that the GIVENS are the amounts already told in the instructions (e.g. EXPENSES = .04). Students will see that the boxes for GIVEN values are all framed in black. Ask students to enter the GIVENs on their sheet, while teacher enters them on viewgraph.

3. Entering varying amounts: Elicit a value for MILES and simulate the computer to compute the COST. Have students enter the values on their worksheet.

4. Assignment sheet: Students should always remember to check the instructions window. Sometimes they will be asked to go on to another problems, and sometimes they will be asked to complete a section on the assignment sheet. Briefly review how to fill out the assignment sheet.

5. Topic completion: When students finish a topic, the menu will reappear. Students should select a new topic if time permits.

4. Pair Assignment

1. Assign pairs. Students in Groups A & B should work at the computers and students in Groups C & D should do their seatwork. Tell students that the Groups C & D students will use the computers tomorrow. **STUDENTS SHOULD ONLY WORK WITH THEIR ASSIGNED PARTNER.** If the partner is absent, the student should work alone. Computer data will be collected for the assigned partners, and also the computer will keep track of their work by ID #.

LESSON 3: USING GRAPHS

Goals:

1 Investigate variable relationships through the use of word-graphs and tables

2. Answer questions about natural situations using intuitive means such as guess-and-test, and tables of values

3. Become familiar with computer operation

4. Become familiar with the semantics of graphs and tables

Activities:

1. Review use of tables. Using one of the domains from earlier in the week, teacher identifies output variable and has students identify input variables. After developing a word graph, make a chart anchoring all but two of the variables. Have students provide values and help teacher do math to construct a table. Ask several questions about the relationships evidenced in the table.

2. Teacher helps students identify output variables in several situations. Students practice formulating problems for translation into word graphs distinguishing between the output and input variables.

3. Discuss graphing. Review the placement of variables and method for plotting points, and the meaning of various graphs.

4. Groups C & D complete Computer Assignment #M-1 (in pairs), and groups A & B work on Seat Assignment #M-2.

LESSON 4: TABLES, GRAPHS, TARGET VARIABLES

Goals:

1. Investigate variable relationships through the use of word graphs and tables
2. Solving simple quantitative discovery-type, and achievement-type problems using informal means, like guess-and-check, table-lookup, and 2d graph reading.
3. Extend a given graph/table by estimating or predicting further values.

Activities:

1a. Review situation modeling:

Abstract situation: a school subsidized trip to Sacramento to learn about state government. Students identify variables and construct word graph; possibilities include:

bus-rental + guide-booklet-cost – subsidy = total-cost

1b. Review table and graphs: (Using review sheet for Day 4)

Word equation is: (miles * mile-rate) + (days * day-rate) = total-cost

Students complete table using a calculator (Part A of review sheet).

Based on the table, students answer the specific questions in Part B of the review sheet: 1 & 2 are questions about the cost of various mileage combinations; 3 & 4 are about relationships between variables.

On blackboard or viewgraph, construct a graph for purpose of reviewing graphing lesson:

```
        +---------------------------------+
        | |   |   |   |   |   |   |   |   |
       300+---+---+---+---+---+---+---+---+
        | |   |   |   |   |   |   |   |o  |   (Based on data in table).
 Total  | |   |   |   |   |   |   |   |   |
  cost 275+---+---+---+---+---+---+---+---+
        | |   |   |   |   |   |   |   |   |
        | |   |   |   |   |   |   |   |   |
       250+---+---+---+---+---+---+---+---+
        | |   |   |   |   |  o|   |   |   |
        | |   |   |   |   |   |   |   |   |
       225+---+---+---+---+---+---+---+---+
        | |   |   |   |   |   |   |   |   |
        | |   |   |   |   |   |   |   |   |
       200+---+---+---+---+---+---+---+---+
        | |   |   |   |   |   |   |   |   |
        | |   |   |   |   |   |   |   |   |
       175+---+---+---+---+---+---+---+---+
        | |   |  o|   |   |   |   |   |   |
        | |   | o |   |   |   |   |   |   |
       150+---+---+---+---+---+---+---+---+
        | |  o|   |   |   |   |   |   |   |
        | |   |   |   |   |   |   |   |   |
       125+---+---+---+---+---+---+---+---+
        | |   |   |   |   |   |   |   |   |
        | |   |   |   |   |   |   |   |   |
        +-+---+---+---+---+---+---+---+---+
         100 200 300 400 500 600 700 800 900

                  Miles
```

2. Referring back to some of the questions asked in the review activity, the teacher introduces the term "target variable", having students circle the target variable for each question. Note that while the word graph remains the same, our interest can shift to different variables, i.e. we have different "target variables" depending on what we want to find out about.

On review sheet: Part B, question 1, has TOTAL-COST as a target variable, because we are asking how much the rental will cost based on a certain distance. Part B, question 2, has MILES as a target variable because we want to know how far the cousin can drive and stay within a set cost.

3. Continuing on with the above problem, the teacher introduces the notion of finding optimal values when a table doesn't have exactly the right value in it.

On the computer, this means moving from seeing how variables relate to each other in a general sort of way, to achieving a known and desired value.

E.g. if the number of days (3) and the day-rate (34) stay the same, how far can the cousin drive and spend exactly 175 dollars?

There are two methods that are used (in seatwork and on the computers) for finding an answer:

i. generate new lines for the table, and "zero in" on the desired number for the TARGET VARIABLE:

E.g. We know that 266 miles gives 160.52 dollars, so we can keep trying higher numbers of miles until we get a number that works. (Answer: 331.81 repeating or about 332 miles)

ii. draw lines on the graph to estimate an answer:

To demonstrate: First connect points already on graph to make a line showing the relationship of miles and total-cost (all other numbers are set).

Then show interpolation from the TOTAL-COST desired to the correct number of MILES by drawing lines from the y-axis to the slanting line, then down to the x-axis.

Once one problem has been demonstrated, call on students to solve similar problems in the same situation:

- How far can the cousin drive and spend exactly $190? $250?
- How much will it cost your cousin to drive 600 miles? 350 miles?

4. Groups A & B complete Computer Assignment #M-3 in pairs; Groups C & D work on Seat Assignment #M-4.

- Computer Assignment #M-3
    1. Solving problems with tables and graphs.
    2. Where students are starting fresh on a new topic, they will also be selecting variables and constructing word graphs.
- Seat Assignment #M-4: Given situations, students model situation, identify relevant target variable, and use tables and graphs to solve problems.

LESSON 5: TRANSLATING WORD GRAPHS TO WORD EQUATIONS AND SYMBOLIC EQUATIONS

Goals

1. Investigate variable relationships through the use of word-graphs and tables
2. Answer questions about natural situations using intuitive means such as guess-and-test, and tables of values
3. Extend a given graph/table by estimating or predicting further values.
4. Evaluate students' ability to perform activities specified in week's goals

Activities

1. ALTERNATIVE REVIEW ACTIVITY. Teacher presents an abstract situation, e.g. buying ice cream with various toppings and various scoop sizes. Students model the situation, construct word graph, table, and graph, and answer questions using tables and graphs.

"You are going to Puffin's Frozen Yogurt Shop for dessert after watching Samohi's "A Midsummer Night's Dream" and want to make sure you can afford the concoction you will order. What do you need to think about when you are estimating your yogurt cost? (perhaps list on board student suggestions):

Size of yogurt, number of toppings, discount coupons, total cost

"Now let's make a word graph of this situation:

Size-Price #-of-topping Topping-Price Discounts -> Total cost

"Let's make a table of the situation. Give me suggestions for:

"size-price	$1.00
"number of toppings?	2
"Price for each topping?	.50
"Discounts	.50

Fill in table with student input:

Size-Price	#-of-toppings	Topping-Price	Discounts	Total cost
1.00	2	.50	.50	1.50
32.00	3	.50	0	3.50
12.50	1	.75	.65	2.60

Now tell students that SIZE-PRICE, TOPPING-PRICE and DISCOUNT will stay the same. Ask for input for #-of-toppings and calculate total cost for several values.

Size-Price	#-of-toppings	Topping-Price	Discounts	Total cost
2.00		.50	.50	
	0			1.50
	2			2.50
	5			4.00
	1			2.00

Graph this:

```
                  8 |
                  7 |
                  6 |
 # of toppings    5 |                              *
                  4 |
                  3 |
                  2 |             *
                  1 |        *
                  0 |__*_______________________________________
                    1.50 2.00 2.50 3.00 3.50 4.00 4.50 5.00 5.50
                                 Total Cost
```

Using the graph, answer such questions as:

"How much would 3 toppings cost?" ($3.00)
"How much would 8 toppings cost?" ($5.50)
"If you had $5.00, how many toppings could you buy? (7 toppings)

2. Teacher briefly notes that algebra is another technique for answering questions, and today the class will work on translating word-graphs into equations so that students can use the equations to solve problems.

3. Using the word-graph from # 1 above, the teacher begins introducing translation process STARTING WITH with terms being added or subtracted.

E.g.,

Size-Price #-of-toppings Topping-Price discounts -> Total cost

Ask whether the toppings are added or subtracted from the bill, and whether the discount is added or subtracted; fill in these two operators.

Size-Price + (#-of-toppings Topping-Price) – discounts = Total cost

— Using several examples, students identify whether term is being added or subtracted from equation, e.g.,

"Would a tip for the person behind the counter be added or subtracted from the total cost?" (added)

"Would a daily special discount on your flavor be added or subtracted from the total cost?" (subtracted)

4. Teacher goes on to identify terms being multiplied, e.g.:

   Size-Price + (#-of-toppings * Topping-Price) – discounts = Total cost

   - Teacher should try to explain the distinction between adding and multiplying in several different ways, e.g.:
     - Multiplication as cumulative addition (one topping @ .50 is for strawberries, one topping @ .50 is for M&Ms, one topping @ .50 is for nuts)
     - When only one topping is being bought, it is like adding the cost of a single topping in. But, when you want more than one topping, you have to figure the price for each one, and then find out how many you are ordering to get the total amount that this will add to your bill.

5. Guided practice: Students work several translation problems on their own. Teacher provides situation and word-graph and then checks problems with whole class.

   - Situation: Buying new outfit at Santa Monica Place.

     WORD GRAPH:

     Shoes socks pants shirt gift-certificates -> outfit cost
     Shoes + socks + pants + shirt – gift-certificates = outfit cost

     *** Note that there are no multiplied word variables

   - Situation: Buying food for an overnight camping trip.

     WORD GRAPH:

     #-of-campers oatmeal dinner trail-mix -> group-food-cost
     #-of-campers (oatmeal + dinner + trail-mix) = group-food-cost

     *** Note that the total food cost for the day is multiplied by the of people number you are buying for.

6. Groups C & D, in pairs, complete Computer Assignment #M-3; groups A & B work on Seat Assignment #M-4.

LINEAR EQUATION UNIT: SECOND AND THIRD WEEK ACTIVITIES

SOLUTION STRATEGIES

Theoretical concepts:

1. Equations are more precise models of real world situations than word graphs.
2. Equations should be approached by first establishing a solution strategy.
3. Problem solving skills can be taught on several cognitive levels: strategic goals, operation goals, number of skills.
4. There are multiple strategies for solving the same problem.
5. Several skills are prerequisite to problem solving, e.g., knowledge of the order of operations and handling signed numbers.

Behavioral Goals:

1. Students will be able to translate a real-world situation into an equational representation.
2. Students will be able to solve equations for desired variables:
    a. Students will be able to specify appropriate variables: step goals for problems with one or two instances of the variable, constants, and coefficients; of the form: $a - bx + c$ and $ax + bx = c$ and $a = b(x + c)$
    b. Students will be able to solve problems of the form: $a = bx + c$ and $ax + bx = c$ and $a = b(x + c)$
    c. Students will be able to use the correct "order of operations" in solving problems
    d. Students will be able to combine signed numbers correctly.
    e. Students will be able to identify constituents of an equation: target variable, constants, coefficients.
3. Students will be able to interpret symbolic answers in terms of the original problem.

Tools:

Modeling/equation solving software
Work sheets
Class lectures
Manipulatives

Pedagogy:

1. Equation writing and solution is tied to real-world situations for motivational reasons, and because translation of problems into equations and answering questions using equations is a primary goal, whereas symbol manipulation is a subsidiary goal.

2. We are trying to implement a general rule of permitting students to focus on one, or a few activities, while still having them see how the specific activity fits into the overall context of mathematical thinking, thus maintaining situated learning.

3. Along with maintaining situated learning, we are implementing he principles of scaffolding and fading, i.e., having the student do one, or a few activities, while the tutor does the rest, with increasing amount of activity being turned over to the student and the tutor fading out.

4. To encourage students to plan problem solutions, they are introduced to equation solution through the specification of strategic goals in problem solving. They are not required to manipulate symbols at this introductory period, so that they can focus entirely on planning. We hope this will not only encourage strategic planning, but reduce the stress involved in having to apply several different types of math processes at once.

5. As strategic planning becomes instantiated, students are introduced to a lower level cognitive goal, specifying solution operations. At the outset, operations are specifically tied to goals.

6. The process for moving from a real-world problem into an equational representation is broken down into the following steps: (i) formulating a problem into a word graph; (ii) translating the word graph into a word equation; (ii) symbolically representing the word equation; (iv) substituting values.

7. Although the tutor models the translation of a real-world problem into an equational representation, and we will have students try to write equations, the tutor currently provides no coaching for problem translation. (The reason for this is that problem translation is a heuristic skill that has not been well-articulated so that a teacher could discuss it, or a computer could automate it.)

LESSON 6: SIMPLIFY (GOALS, MATH), REDUCE (GOALS)

Goals:

1. Review: Wordgraphs; target variable; word equations; signed numbers
2. Introduce: SIMPLIFY (goals/operations/math levels); REDUCE (goals level)

Activities

1. Review word graphs. Teacher describes the abstract situation, "Knowing the profit from a school dance":

> The senior class is holding a dance to help pay for yearbooks. It's going to be a sock-hop in the gym, using taped music. The students are trying to figure out their profit depending on what they charge and how many tickets they sell.

Asking for student help with the variables, the teacher generates an appropriate wordgraph with student input:

Tickets sold Charge > Profit

2. Teacher translates word graph to word equation, with student input; and then into symbolic equation.

Tickets sold * Charge = Profit

$t * c = P$

3. Teacher substitutes values, the seniors sell 100 tickets, and charge $4.00 a ticket. She circles the target variable "profit" and then asks students what has to be done to find out the value of the target variable. Label the goal and process of "combining numbers" as SIMPLIFYing:

$100 * \$4 = P$

Teacher asks students to solve the equation, and then checks the answer on the board.

Teacher and/or students generate several more equations, using different values for "tickets" and "charge". (Each time, the teacher asks students for suggestions on how many tickets will be sold, and what the charge should be.) Students solve each new equation, and teacher checks it on the board.

4. Teacher rewrites original word equation, and changes the target variable to one of the input variables, circling the target variable. She elicits from the students that the target variable is multiplied by a number, and then explains that to find out the value of the target variable, we need to get the target variable by itself, this goal will be called REDUCE.

tickets * Charge = profit

$t * C = p$

$20(C) = 100$

- Teacher labels the goal and does the arithmetic

  REDUCE > $C = \frac{100}{20}$

- Ask students what the next goal would be (SIMPLIFY), and finish solution.

  SIMPLIFY > $C = 5$

- Explain that to check the answer it should be substituted back into the original equation. Teacher substitutes value and asks students to do arithmetic

4. Use another situation involving rate of speed * Time = distance:

   You're going to a rock concert in Santa Barbara. Your average speed is 45 mph, and the distance is 90 miles. How long will it take to get there?

   The teacher writes the equation, substitutes values, then asks students for the target variable. The teacher circles the target variable and asks students for the goal (REDUCE)

   $s * T = d$
   $45\,(T) = 90$

   Students specify REDUCE teacher does arithmetic 90/45 = T
   Students specify next goal, SIMPLIFY, teacher does arithmetic

- Define coefficient – a number multiplied to a variable

5. Teacher notes that multiplication can be represented in many different ways: 20 * C 20 x C 20(C) 20C

- Using the equation from "4" above, have students circle the coefficient, then represent the multiplication in several different ways

6. Teacher briefly reviews how signed numbers are combined. Display chart which can remain visible during the class:

   To combine numbers with same signs, add the numbers. The answer has the sign of the numbers:

   $-6 \quad -5 \quad = 11 \qquad\qquad +3 \quad +4 \quad = +7$

   To combine numbers with different signs, subtract the smaller number from the larger. The answer has the sign of the larger number:

   $10 \quad +2 \quad = 8 \qquad\qquad +14 \quad -4 \quad = +10$

   Multiply or divide numbers with like signs and the answer is positive:

   $5 \quad * \quad -4 \quad = +20 \qquad +3 \quad * \quad +4 \quad = \quad +12$

   Multiply or divide numbers with unlike signs and the answer is negative:

   $+5 \quad * \quad -5 \quad = \quad 25$

11. Teacher asks the class to do the following problems. Students do problems and teacher checks answers:

speed * time = distance

1. You're going to a rock concert in San Diego. This time the speed is 50mph and the distance is 100 miles. How long will it take to get there?

Teacher writes on board: 50 * t = 100

She asks students for the target variable and circles it.

She asks for first goal: ___________

(If students say SIMPLIFY, ask them to explain, trying to elicit that there are not two numbers multiplied together but a number and a variable. So the goal is REDUCE.)

Teacher does math: $t = \frac{100}{50}$

Teacher asks for next goal: ________

Students do math.

The teacher uses the same procedure with the next problem: This time a group is going to a rock concert at the Orange County center. They know it will take one hour to get there, and the distance is is 35 miles. How fast are they driving?

12. Teacher briefly introduces goal level software – SEE LESSON ON FOLLOWING PAGE.

13. Groups A & B work in pairs on computer assignment #M-6 (15min): Modeling situations and solving equations using the goals SIMPLIFY and REDUCE.

14. Class assignment (15min): All groups are responsible for Assignment # M-5 (Seat) – "Combining signed numbers." (Marcy Sheets AA31–32). Groups C & D are also responsible for Assignment #M-7 an exercise involving specifying goals for problems requiring SIMPLIFY and REDUCE.

VOCABULARY: target variable, coefficient, SIMPLIFY, REDUCE

EQUATION SOLVING SOFTWARE: INTRODUCTORY LESSON FOR STUDENTS

1. **Distribute student worksheet.** The student worksheet will have a picture of the modeling software on side one, and of the modeling and equation software on side two.

2. **Tell students that the login procedure is the same.**

3. **Show viewgraph-A of typical week 2 screen showing the modeling window. Ask** students to read aloud the situation. (The situation is the same as was used for the computer introduction, buying a car.) Variable names will already be filled in the word equation, and the first operator will also be filled in.

```
______    ___   _________   ___   _________   ___   ____

|MILES|   |*|   |EXPENSES|  | |   INSURANCE   |=|   |COST|
------    ---   ---------   ----  ---------   ___   ----
```

- Point to the word equation and discuss the relation of the variables.
- **Ask students to write in the missing operator.** Tell them that on the computer, they will CLICK, TYPE, and hit RETURN to enter the operator.

4. **Point to the Symbolic Equation.** Tell students that they will be choosing letters to stand for the words. **Have them write in the letters.**

5. **Show viewgraph-B of modeling windows, with Assignment M-6 problem displayed.**

- Ask students to read instructions
- Ask students to identify TARGET variable. **Have students circle target variable in the instruction window.**

6. **Show viewgraph-C of equation solving window.** Point out various features of equation solving window (e.g. Help, Goal names, Comment window, etc.). Tell students that they will type in the goal for each step of the problem, and the computer will do the math.

- **Have students write the goal for the first step in the problem next to the "Current Goal" prompt.**

7. When students have finished a problem they should click on "Answer OK".

- **Have students circle "Answer Ok" on their sheet.**

8. After the computer checks the problem the student will fill in his worksheet. **Review worksheet questions with students, showing sample answers from the problem on their sheet:**

What was the target variable? (expenses)

Explain your answer in a short sentence. (The expenses for the car were 5 cents a mile.)

9. **Remind students that they can choose any topics they want, and that they should complete as many topics (and problems) as they can.**

LESSON 7: ISOLATE (GOALS), REDUCE (OPERATIONS, MATH)

Goals:

1. Review: SIMPLIFY; REDUCE
2. Introduce: REDUCE at operations and math level, ISOLATE at goals level

Activities:

1. Review SIMPLIFY, REDUCE:

   - Describe an abstract situation for raising money from a football game: When you figure out how much money a football game will raise, you need to think about how many people will attend and the ticket cost.
   - Teacher generates a word graph and then a word and symbolic equation with student input: e.g., Ask, "If we want to know how much money we'll raise, what do we need to think about?"

#PEOPLE TICKET-COST > MONEY-RAISED

PEOPLE * TICKETS = MONEY

$$p * T = m$$

   - Substitute values, identifying "ticket cost" as target variable; and have students specify solution goals while teacher does math.

$$500 * T = 1500$$

$$\text{REDUCE} > \frac{500T}{500} = \frac{1500}{500}$$

$$\text{SIMPLIFY} > T = 3$$

   - Ask students what had to be done to REDUCE the "500T". Explain that whenever a variable has a coefficient, we get rid of it by dividing by the coefficient and thus cancelling it out. HOWEVER, if one side of the equation is divided, to keep the equation even, the other side must also be divided.
   - Teacher gives several similar problems, first modeling their solution, then having students solve them. Examples to use:
     - If money raised was $2000 and the number of people who attended was 750, what must the ticket price have been? ($750*T = 2000$)
     - If ticket price was $4 and money raised was $2200, how many people must have shown up? ($p*4 = 2200$)
     - If the money raised was only $1500 and ticket price was still $4, how many people come? ($p*4 = 1500$)

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2. Teacher rewrites the original word equation and adds another term to the equation, e.g. "security". Teacher underlines the target variable and asks students what has to be done to find out the value of the target variable, ticket cost? (REDUCE it from its coefficient and ISOLATE it from the the number that is subtracted from it).

#people * ticket cost – security = Money-raised; $p * T - s = m$

$$500T - 100 = 1500$$

— Teacher explains that often it is easier to begin by isolating the target variable from any term added or subtracted to it, and then from the coefficient. Teacher asks students to specify goals, and she does the arithmetic, with student help.

	$500T - 100 =$	1500
ISOLATE >	$500T - 100 +$	$100 = 1500 + 100$
SIMPLIFY >	$500T =$	1600
REDUCE >	$500T/500 =$	$1600/500$
SIMPLIFY >	$T =$	3.20

— Check by substituting value back into original equation

3. Teacher provides the following examples and ask students to circle the coefficients and/or terms connected to the variable:

a) # of renters * Fee = video store sales

Target variable is "Fee", circle the coefficient:

$100(F) = 2{,}000$

b) renters * Fee + membership = video store sales

Target variable is "fee", circle the term connected to the variable:

$100F + \$10 = \$2{,}000$

c) # of miles run * pledges per mile = amount of money you raise

Target variable is "# of miles you run", circle the coefficient:

$M(25) = 500$

d) miles * pledges – entry fee = money raised

Target variable is "miles", circle the term connected to the variable:

$M(25) - 50 = 500$

4. Teacher briefly review meaning of SIMPLIFY and **ISOLATE and REDUCE**.

- ISOLATE – means to remove a term added or subtracted to a variable, so that it will stand alone
- REDUCE – means to remove a coefficient from the variable, so the variable will stand alone
- SIMPLIFY – means to combine numbers

5. List these three goals on the board, and then ask students to specify the appropriate goals for the following problems:

renters * fee – salaries = video store Sales

r * f + m = S   100 * $2 – $44 = S

- Explaining that the store owner wants to know what his sales will be, ask students to underline the target variable, the one they want to know about (SALES).
- Ask if there's anything there with it, i.e. does it need to be ISOLATED or REDUCED. Since it's already alone, what is our goal (i.e., what needs to be done to find out how much money the video place has made? (SIMPLIFY)
- Check answer in original equation

In the next problem, explain that the owner wants to know how many renters he'll need to earn $4000. Ask students to underline the target variable, the variable they want to find out about. Ask them if it's alone, if not, what strategy do they have to use to begin to solve the problem?

R($2) – $44 = $4,000

ISOLATE > $2R = 4000 -44

- Ask for the next step (REDUCE or SIMPLIFY) and illustrate its results (Note that students could use either strategy and on the computer they can try both and see which they prefer.):

REDUCE: $R = \frac{\$4000 - 44}{\$2}$ -OR- SIMPLIFY: R(2) = 3956

- Pursue the solution, having students specify the next goal
- Use the following problem based on selling tickets to earn a profit. Ask students to help write the word graph and then the word equation:

ticket price * #kids – expenses = profit

Teacher labels target variable "#kids" then: t * K – e = p

Teacher substitutes values: $4 * K – $25 = $375

4K – 25 = $375

- Students specify goals and teacher does math with students' help for SIMPLIFY and REDUCE

ISOLATE > $4K = 375 + 25$

SIMPLIFY > $4K = 400$

REDUCE > $\frac{4K}{4} = \frac{400}{4}$

SIMPLIFY > $K = 100$

Check: $4(100) - 25 = 375$

6. Groups C & D work in pairs on computer assignment #M-9: modeling and equation solving at goal level using SIMPLIFY, REDUCE, & ISOLATE, then they work on Seat assignment M-8.

7. Groups A & B work on Seat Assignment #M-8: Solving problems requiring REDUCE and SIMPLIFY at the goals, operations, and math level; and Seat Assignment M-10: Specifying goals for problems requiring SIMPLIFY, REDUCE, and ISOLATE.

VOCABULARY: ISOLATE

LESSON 8: ISOLATE (OPERATIONS, MATH)

Goals:

1. Review: SIMPLIFY; REDUCE; ISOLATE
2. Introduce: ISOLATE at operations and math level
3. Introduce tutor software for combined goal-operations problem solving

Activities:

1. Check some of the problems from seat assignment 4 as a review of SIMPLIFY, REDUCE, and ISOLATE at the goal level, and SIMPLIFY and REDUCE at the operations and math level.

2. Introduce the operations and math needed to ISOLATE. With the video rentals situation, use the following equation requiring ISOLATE, with the target variable RENTERS, e.g.

Fee * Renters – Salaries = Profit

$2 * R – $200 = $4,000

Elicit next step from students:

ISOLATE > 2R – 200 + 200 = 4,000 + 200

Ask students what had to be done to move the "200" to the otherside. Explain that to ISOLATE a variable with a term added or, subtracted from it, do the opposite, i.e., in this case, add "200" so the terms will cancel each other out. HOWEVER, to keep the equation equal, whatever is added (or subtracted) from one side, must be added (or subtracted) from the other side.

Complete the problem solution, having students first specify the goal, then help with the math, as follows:

$$2R - 200 + 200 = 4{,}000 + 200$$

SIMPLIFY >

$$2R = 4{,}200$$

REDUCE >
Operation needed for
REDUCE is divide each
side by 2 (/ 2)

$$\frac{2R}{2} \quad \frac{4{,}200}{2}$$

SIMPLIFY >

$$R = 2{,}100$$

3. Work through second problem, getting student input for the solution:

This time, suppose the fee for renting videos is still \$2 per video, but salaries have increased to \$400. If profit dropped to \$3,600. How many renters were there? The equation is:

Fee * Renters – Salaries = Profit

$2 * R - 400 = 3{,}600$

ISOLATE >
Operation:
+ 400 $\quad 2R - 400 + 400 = 3{,}600 + 400$

SIMPLIFY > $\quad 2R = 4{,}000$

REDUCE >
Operation:
/ 2

$$\frac{2R}{2} \quad \frac{4{,}000}{2}$$

SIMPLIFY > $\quad R = 2{,}000$

4. Explain giving operations on the computer:

- reading the problem and picking the target variable is the same as when doing only the GOALS
- to solve problems, first give the GOAL as before. Then when the computer highlights the line that says "Current operation:", type the OPERATION you need to carry out the GOAL.
- OPERATIONs given to the computer have three parts:
    1. +, –, * or / to show whether something is to be added to, subtracted from, multiplied by or divided into each side, respectively.
    2. a space
    3. the number that's supposed to be added, subtracted, multiplied or divided

Thus, dividing each side by 2 is given by "/2".

Adding 400 to each side is "+ 400".

NB: "/2" won't work (no space), nor will "+400".

- After the OPERATION is given, the computer will write out the next step of the equation and ask for the next GOAL again.
- When a solution has been found, checking the answer is just as in previous computer sessions.

5. Groups A & B will each do Computer assignment M-11 (15min each) and then begin Seat assignment #M-12. Groups C & D will work on Seat assignment #M-12.

Computer Assignment #M-11: Modeling and equation solving using SIMPLIFY, ISOLATE, REDUCE with the goal and operations level software.

Seat Assignment #M-12: problem solving using SIMPLIFY, REDUCE and SOLATE at goals, operations, and math levels.

LESSON 9: THE EQUALITY OF AN EQUATION

Goals:

1. Review: SIMPLIFY, REDUCE, ISOLATE
2. Introduce: Concept of equation as an equality

Activities:

1. Alternate Review Activity to review problems requiring SIMPLIFY, REDUCE, ISOLATE at goals, operations, and math levels, e.g.,

Ex: Total number of students buying yearbooks: 4 classes each have 30 kids, all but 4 in each class buys the book:

4(30 – 4) = # kids buying

SIMPLIFY: Goals: SIMPLIFY -> 4(26) = # of kids

SIMPLIFY -> 104 = # of kids buying yearbooks

Ex: Each of the 11 members of the swim team sold the same number of carnations on last Valentine's Day. If they made $440, how much money did each swim team member bring in?

11 * amount = 440

REDUCE: /11 11a/11 = 440/11

Do division a = $40

Each team member sold $40 worth of flowers.

Ex: On your trip to Hawaii this winter, you have decided that you don't want to make the trip from Maui to Oahu that you have already paid for. You don't remember what the total cost of the original ticket was, but the trip to Maui was going to cost you $516. You end up getting a refund check in the mail for $172 for the unused part of your ticket (the hop to Oahu you didn't take). How much must that original total ticket cost?

Total Price – refund = Maui trip

P – $172 = $516

ISOLATE: +172 P – $172 + $172 = $516 +$172

Do addition P = $688

2. Use scale or some other manipulative to portray that an equation is an equality.

Explore with students what happens as different quantities are added and subtracted:

"Equations works like balance beams (see-saws) and the place where they balances is the equal sign. In order for the equation to remain an equation, you MUST

FOLLOW THE RULE: whatever is done to one side of an equation must be done equally to the other side, because the "=" means that each side must equal the other.

BRIEFLY refer back to REDUCE and ISOLATE problems previously completed in class and remind students that whatever was done to one side of the equation had to be done to the other: this is especially visible at the operations level. Give an example and get student input on what to do, e.g.:

At a dance:
Tickets sold + food sold = profit T + F = P,

— Present situation: "We made $500 on the dance. We know we sold $300 worth of tickets; how much food did we sell?"

—Write equation on the board:

$$300 + F = 500$$

Talk students through the problem, e.g.:

- — Ask students how they think they could determine the food sales (Hopefully they will intuitively figure 500 – 300 and/or specify ISOLATE.)
- — Ask, "If we subtract "300" from one side of the equation, what do we have to do on the other side" (Also – 300), Why did we do it? Because this keeps the equation even.

Write out problem, adding goal and operation labels:

$500 = 300 - F$

ISOLATE, –300 500 $-300 = 300 - F - 300$

SIMPLIFY $200 = F$

Talk students through another problem, e.g., Joining a health club.

"Prices for health clubs are very competitive in the area. One club is offering the following deal: a one year membership for $444. They claim that this is a special low price, lower than their normal rate of $36 a month. How much of a deal are they giving you?

$$12 * \text{monthly-rate} = \$444$$

$$12 * m = \$444$$

REDUCE: /12 $$\frac{12 * m}{12} = \frac{\$444}{12}$$

$$m = \$37$$

Again, highlight that with dividing the entire side of the equation must be divided, with adding/ subtracting, the term being added/subtracted is added only once to each side of the equation.

If there is time, DEMONSTRATE that doing different things to the two sides of an equation makes it NO LONGER AN EQUALITY.

$3 * X = 15$ <- WE KNOW THAT X = 5)

divide only one side by 3. . .

$X = 15$ <- but we knew that X had to be 5 from before

CHECK by SUBSTITUTING! $3 * 15 = 15$

$45 = 15$ <- not possible

3. Help students complete the "Money Bags" worksheet.

4. Groups C & D complete Computer assignment # M-11, and groups A & B work on Seat assignment #M-12.

LESSON 10: COLLECT (GOALS, OPERATIONS, MATH)

Goals:

1. Review SIMPLIFY, REDUCE, ISOLATE (g/o/m levels)
2. Introduce COLLECT

Activities:

1. Review SIMPLIFY, REDUCE, ISOLATE. Teacher presents a high school car wash situation. With student input she generates the word & symbolic equation, and then the teacher substitutes values, designating the target variable, "cars":

fee * Cars – expenses = profit

$f(C) - e = p$

$4C - 10 = 90$

- Teacher asks: What do we want to know about? she underlines target variable
- Teacher asks students for goal, then operation, then math e.g.: What's our first step? (ISOLATE) What do we have to do to ISOLATE? (+ 10) How do we do that? (+10 to both sides, etc.)
- Pursue problem to its solution, then check answer in original problem.

2. Introduce COLLECT:

a) Teacher generates situation involving manipulatives (e.g. different types of fruit, or color-coded groups of students), students physically "collect" themselves into appropriate groups to solve problem.

b) Teacher rewrites word equation from "1" above, to include spray wax jobs. Cars is still the target variable. We want to find out how many Cars will have to be washed and waxed to make a profit of $440.00. Underline target variable:

$f * C + (w * C) - e = p$

- Teacher substitutes values: $\$4 * C + (\$5 * C) - 10 = \$440$

$4C + 5C - 10 = 440$

Ask students for next step; explain why ISOLATE and SIMPLIFY are not appropriate yet; reinforce any responses about adding the two instances of the variable.

ISOLATE – requires there be only ONE instance of the variable
SIMPLIFY – combines NUMBERS

Introduce the term COLLECT – combine like variables. Explain that the coefficients of the variable are combined.

- Continue on with the problem, eliciting goals, and explaining operations and math.

	$\$440 = 4C + 5C - 10$
COLLECT >	$\$440 = 9C - 10$
ISOLATE; +10	$\$440 + 10 = 9C - 10 + 10$
SIMPLIFY	$\$450 = 9C$
ISOLATE; /9	$\frac{\$450}{9} = \frac{9C}{9}$
SIMPLIFY >	$50 = C$

— Ask students to substitute value of "C" into original equation to check results. (Refer to Order of Operations chart as necessary.)

$\$450 = (50 * \$4) + (50 * \$5)$

3. Use the following two situations to provide more practice using COLLECT. Students should help establish word graphs and the translation into word equations, specify goals and do math.

During the presentation emphasize the conditions that dictate a specific strategy step, and the occasions when a choice of strategic steps is available.

a) – Situation: Your group is going to a banquet. The total cost (C) of the banquet to the group depends on the total number of people that are going (P), the cost of food per person (F), and the admission cost per person (A).

- The equation is: $F*P + A*P = C$
- Specific problem example:

Assuming your group has $600, admission price is $5, and the cost of food per person will be $20, how many people from your group can go to the banquet?

b) – Situation: You want to collect energy in atomic particles in a physics experiment. The particles are generated by a cyclotron, and pass through three energy boosting rings before being captured in a lead box. The total energy (E) you capture depends on the number of particles (N) you capture, the energy given to each particle by ring 1 (A), the energy given to each particle by ring 2 (B), the energy given to each particle by ring 3 (C).

- The equation is: $A*N + B*N + C*N = E$
- Specific problem example:
Assuming you want to collect a total of 10000 units of energy, and that ring 1 gives each particle 2 units of energy, ring 2 gives 3 units of energy, and ring 3 gives each particle 5 units, how many particles do you need to collect?

4. Groups A & B working in pairs, Computer assignment #M-15, and work on Seat assignment #M-14.

5. Groups C & D, Complete Seat assignments #M-14 and M-16.
Computer Assignment # M-15: Problem solving using
SIMPLIFY, REDUCE, ISOLATE, COLLECT goals & operations level.

Seat Assignment #M-14: Problem solving with SIMPLIFY, REDUCE, ISOL, COLLECT at goals, operations, math level. Seat Assignment #M-16: Problem solving with SIMPLIFY, REDUCE, ISOLATE, COLLECT at goals and operations level.

LESSON 11: DISTRIBUTE (GOALS)

Activities:

1. Review equation solving.
2. Introduce DISTRIBUTE.

Present situation: A computer club wants to raise \$100 for a modem. They meet 8 days each month, and each time they have to pay \$5 cleaning charge for the meeting room. What kind of fee should they charge at each meeting to cover their expenses nd raise \$100?

days(fee – cleaning) = modem

$d\,(F - c) = m$

$8\,(F - 5) = 100$

Ask students what they think is the first step. If inappropriate answers are given, counter by specifying conditions that must be met for the step. If students don't know, suggest completing the multiplication, using the distributive rule. Goal is to DISTRIBUTE. Complete the problem at the goal level, with students specifying goals and teacher doing math.

GOAL LEVEL:

	$8(F\text{-}5) = 100$
DISTRIBUTE >	$8F - 40 = 100$
ISOLATE >	$8F = 100 + 40$
SIMPLIFY >	$8F = 140$
REDUCE >	$\frac{F}{8} = \frac{140}{8}$
SIMPLIFY >	$F = 17.50$

Replace answer in original equation to test whether it's correct.

3. Do another problem using the same equation. This time, the club thinks they can get the modem for a discount, \$80.00. They still meet 8 days, but they've been so neat that the cleaning fee is only \$3.00. Complete the problem with students specifying goals and teacher doing math:

days(Fee – cleaning) = modem

$8\,(F - 3) = 80$

DISTRIBUTE >	$8F - 24 = 80$
ISOLATE >	$8F = 80 + 24$
SIMPLIFY >	$8F = 104$

REDUCE > $\frac{F}{8} = \frac{104}{8}$

SIMPLIFY) $F = 13.00$

4. Use the following situation and do two more problems requiring DISTRIBUTE. Have students help write equation and then specify goals and help perform symbol manipulations when appropriate.

— Problem A
The science class is going on a field trip to Griffith Park Observatory. The students have to pay a lunch fee and a bus fee of $2.00. Twenty-five students are going on the trip. The school is providing $137.50 for the trip. How much can each person spend on lunch?

students (lunch fee + bus fee) = trip cost

$s(L + b) = t$

$25(L + 2) = 137.50$

— Problem B
The school was so pleased that all the students wanted to go on the trip, that they agreed to provide $180.00 for the trip. Now all 32 students are going. How much will each person haveor lunch now?

$32(L + 2) = 180$

5. Groups C & D working in pairs, complete Computer Assignment #M-18 and Seat Assignment #M-17. Groups A & B work on Seat Assignments #M-17 and #M-19.

Seat Assignment #M-17: Problem solving with SIMPLIFY, REDUCE, ISOLATE, & COLLECT at goals, operations, and math levels. Assignment #M-19: specifying goals for all commands.

LESSON 12: DISTRIBUTE (OPERATIONS, MATH)

Activities:

1. Review material from Seat Assignment #17.

2. Introduce DISTRIBUTE at the operations and math level. Use the following situation. Have students help develop word equation. As the problem is being solved the students should specify goals and the teacher should explain the math involved in DISTRIBUTing.

— The football team wants to buy an end of season thank-you present for all the cheerleaders. They decide to buy a corsage and a box of candy for each girl. There are 6 cheerleaders; flowers are \$2.50 each. The team has collected \$45.00. How much can they afford to spend on candy for each girl?

# of girl cheerleaders(flowers + candy) = team money

$$6\,(2.50 + C) = 45.00$$

1st Goal:	DISTRIBUTE	$15 + 6C = 45$
operation:	distribute 6	

Explain that the term outside the parentheses is to be multiplied against each of the terms inside the parentheses; teacher continues on eliciting goals and doing math with students' help.

3. Ask students to do the following problems. Check problems together:

a. What if the team thinks they can get candy for \$4.00 and want to know how much they can spend on flowers, (total team money stays at \$45.00):

$$6\,(F + 4.00) = 45$$

b. What if the team decides to include the cheerleaders' manager, so now there's 7 gifts; they spend \$3.75 for flowers; they collected \$47.25. How much could they spend on candy?

$$7\,(3.75 + C) = 47.25$$

c. What if the team decides to buy thank you presents for all the song leaders, too. If they spend \$3.00 on flowers, \$4.50 on candy, and have collected \$75.00; how many people can they buy presents for? #of people (flowers + candy) = team money

$$P\,(3.00 + 4.50) = 75$$

Note: on the computer, this problem needs to begin by SIMPLIFYING

4. Introduce students to the math level software. Tell students that after specifying the goal for each step in the problem solution, they will be asked to type in the actual next line of the equation. Calculators will be available. Example:

$$2d + 3 = 23$$

Goal: ISOLATE

Equation: $2d + 3 - 3 = 23 - 3$

Students can do as much figuring in their head or with a calculator as they want. For example students could enter the ISOLATE equation, above, in any of the following ways:

Equation: $2d = 23 - 3$

Equation: $2d = 20$

5. Groups AB and CD in pairs, 15 min each set of groups, complete Computer Assignment #M-20. Students individually work on Seat Assignment #M-21.

Computer Assignment #M-20: Problem modeling and problem solving at math level.

Seat Assignment #M-21: Problem modeling and problem solving.

GLOSSARY

ACTIVITY PROFILE—A complete specification for each model-building activity and equation-solving activity whether: (i) it is to be done by the student, (ii) by the tutor, or (iii) not done at all (or done implicitly by the tutor).

ACHIEVEMENT QUESTIONS—Specific questions about a natural situation in which the output variable is known, and the student must infer what the values of other variables must be for the output variable to achieve its desired value. Example: "If total sales = 300 – 3*cost-per-unit, and you want total sales to be 250, what must cost-per-unit be set to to achieve this goal?"

DISCOVERY QUESTIONS— Specific questions about a natural situation in which the output variable is unknown, and the student must infer what it is given the values of other variables. Example: "If total sales = 300 - 3*cost-per-unit, and cost-per-unit is 20, what will total sales be?" ,lp CHECKING AN ANSWER—In solving a symbolic equation, the interpretation of the answer back in terms of the original statement of the specific problem.

EQUATION-SOLVING—Activities that take a specific problem and equational model of an abstract situation (symbolic equation) and make an inference about the value of the target variable in the specific problem. We somewhat arbitrarily divide this into 5 subactivities: (i) mapping the specific problem onto the equational model, (ii) reasoning about goals, (iii) reasoning about operations, (iv) symbol manipulation, and (v) checking an answer arrived at by symbol manipulation against the original specific problem.

EQUATIONS:

SYMBOLIC EQUATION—Equation where all operators and words denoting variables are represented using traditional algebraic syntax: single letter variables, and operators like, *, /.

WORD EQUATION—Equation where operators are in traditional algebra, but variables are English labels. The variables may be just the English labels, or variable boxes. For example:

```
  ------------          ------------    ------------    ------------
 | profit      |        | #students   |  | cost/ticket|  | expenses    |
 |-------------|   =    |-------------|* |------------|- |-------------|
 |             |        |             |  |            |  |             |
  ------------          ------------    ------------    ------------
```

GOAL REASONING—In solving a symbolic equation, the determination of which goal to follow; for example "isolate".

GRAPHS:

WORD GRAPH—Description of a situation where all variables are represented using variable boxes, but operators are not. Graphical version of "y=f(x,z,...)".

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```
 ------------         ------------    ------------    ------------
| profit     |       | #students  |  | cost/ticket|  | expenses   |
|------------|  <--  |------------|  |------------|  |------------|
|            |       |            |  |            |  |            |
 ------------         ------------    ------------    ------------
```

2D GRAPH—A standard graph plotting values of two natural variables against each other. Both axes are labelled with the names of the variables.

MAPPING A SPECIFIC PROBLEM—In solving a symbolic equation, the activity of interpreting a specific problem in terms of an equational model. This breaks down into several subactivities: (i) identifying the variable being solved for in the equation, (ii) identifying the other variables and substituting into the equation values given in the specific problem.

MODEL-BUILDING—Activities that transform a natural-language description of an abstract situation into an equational model that can be used to answer many specific questions. We somewhat arbitrarily divide this into 3 subactivities: (i) formulation, (ii) translation, and (iii) symbolization.

OPERATION REASONING—In solving a symbolic equation, the determination of which operation to apply; for example "+9".

PROBLEM FORMULATION—Process of taking an ill-defined description of a situation into a list of variables that define the situation, possibly also identifying the input and output variables. More specifically, for our purposes, we define formulation as converting a word statement representation of a problem into a word graph.

PROBLEM TRANSLATION—Process of taking a situation that has been defined to the point of a variable list or word statement, and turning it into a word equation.

PROBLEM SYMBOLIZATION—Process of transforming a word equation of an abstract situation into a symbolic equation.

SITUATIONS:

ABSTRACT SITUATION—A "real world" situation that encodes a relationship between two or more natural variables. The situation is abstract in the sense that specific values are not given for the variables. Example: 'Jill has 10 more than 3 times as many marbles as Jim.' is an abstract situation involving 2 natural variables, 'Jill's marbles' and 'Jim's marbles'.

SPECIFIC SITUATION/PROBLEM—An abstract situation where values of some natural variables are given. Example: 'Jill has 10 more than 3 times as many marbles as Jim. Jill has 58 marbles.' is a specific situation. These are typically the kinds of things that are the basis for "problems" in textbooks. A specific situation is described where one or more natural variables are not given values (i.e., unknown), but their value(s) can be inferred from the abstract situation constraints, and given variable values.

SYMBOL MANIPULATION—In equation solving, the actual writing of a new equation that follow from the previous one by some algebraic transformation.

TABLE OF VALUES—A columnar table that encodes numeric values for covarying natural variables in a situation. Each column represents values for a different variable. Each row represents values for different variables at the same time (i.e., each row represents a *specific* situation). Each column is headed by the name of the variable it represents.

VARIABLES:

ANCHORED VARIABLE—A variable which the system has decided will now be protected from change by the student. The student cannot type different values into the variable box representing the variable. An anchored variable will be denoted by a variable box whose border is thicker.

INPUT VARIABLE—In an abstract situation, the variable that is intuitively the causally independent one. The term "independent variable" is thus often used synonymously. For example, "unit price of widgets" is an input variable; "total number of widgets sold" is a variable that (may) causally depend on it. Note, typically, equations of functions that relate variables express the natural output variable as a function of the input variable. In cases were there is no causal dependency among variables, the input variable is simply the one that is algebraically in the input position. For example, in "Sara's age is 5 more than 3 times Bill's age", "Bill's age" is the input variable. Note that it is more difficult to solve for the input variable, when the equation expresses the natural output variable as a function of the input variables, since substitution alone will not work.

NATURAL VARIABLE—A feature of a real world situation that can take on different values. We assume the values can be represented by numbers. Examples: the age of Bob, the net profit of a sale.

OUTPUT VARIABLE—In an abstract situation, the variable that is intuitively the causally dependent one. The term "dependent variable" is thus often used synonymously. For example, "total number of widgets sold" is an output variable; "unit price of widgets" is an input variable that may causally determine it. Note, typically, equations of functions that relate variables express the natural output variable as a function of the input variable. In cases were there is no causal dependency among variables, the output variable is simply the one that is expressed algebraically as a function of another. For example, in "Sara's age is 5 more than 3 times Bill's age", "Sara's age" is the output variable. Note that it is easier to solve for the algebraically output variable, when the equation expresses the natural output variable as a function of the input variables, since substitution alone will work.

TARGET VARIABLE—The variable the student must solve for in an algebraic equation, or the variable for which an equation has already been solved. (E.g., in "unit-price = gross-income/units-sold", "unit-price" is the target variable). Note that target variables are distinct from output or dependent variables. Output variables are causally dependent on input variables. A target variable is one that has been, or should be, solved for algebraically. In general, algebra abstracts away from causality in powerful ways; input and output variables can be targets, at differenttimes.

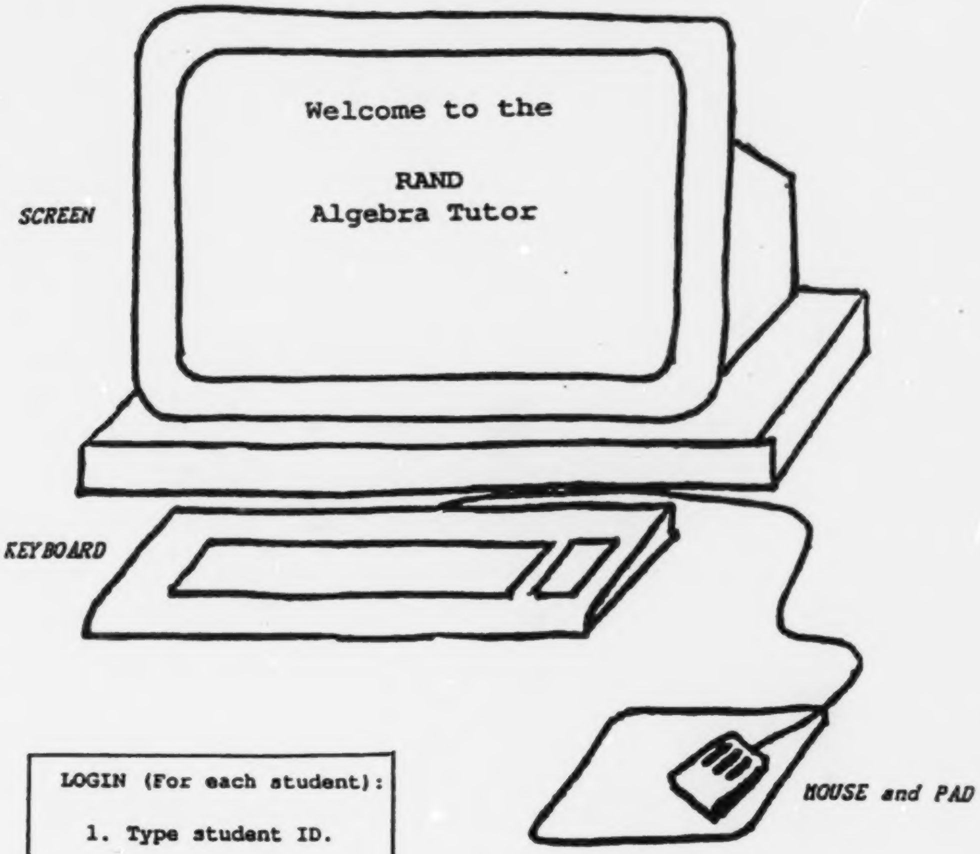
VARIABLE BOX—Somewhat graphical representation of a variable, including its name, and a slot for a numeric value. For example:

```
 ________________
| profit        |
|---------------|
|  1000         |
 ________________
```

WORD STATEMENT—Statement of an abstract situation in English. Example: 'The number of widgets sold is 300 less 2 times the unit cost of a widget'.

WORD STATEMENT PROBLEMS—Problems where enough of the variables of a word statement are turned into constants that new information can be inferred. This inference is the problem. Example: "'The number of widgets sold is 300 less 2 times the unit cost of a widget.' 'If the number of widgets sold was 150, what was the unit cost of a widget?'"

STUDENT MATERIALS


Welcome to the
RAND
Algebra Tutor
SCREEN
KEYBOARD
MOUSE and PAD

LOGIN (For each student):

1. Type student ID.

   Press RETURN.

2. Type first name.

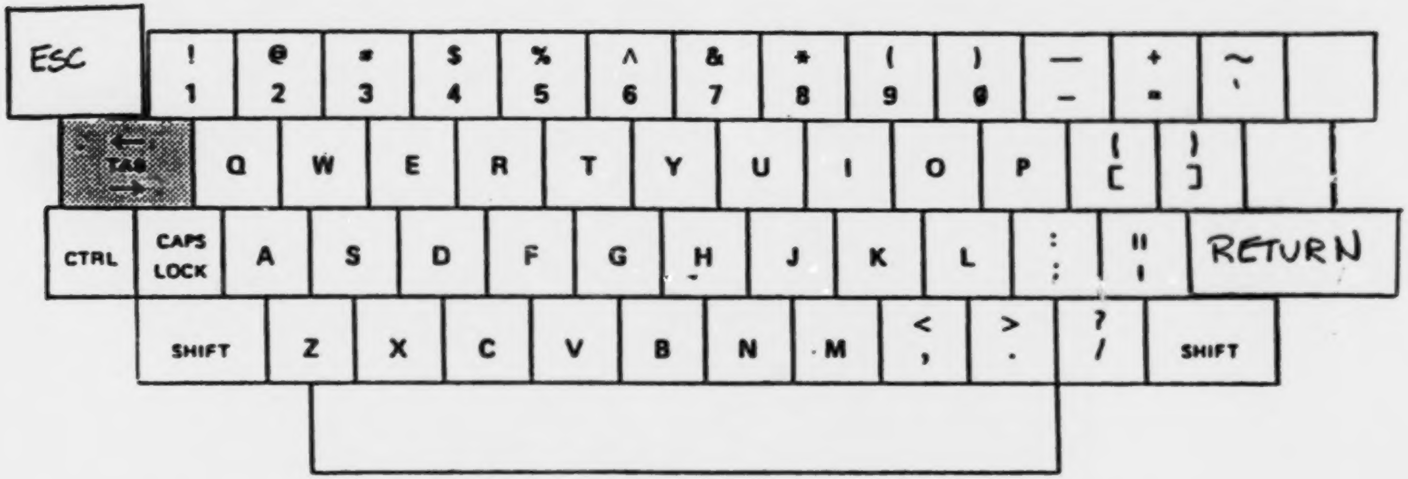
   Press RETURN.

CHOOSE A TOPIC:

1. After you login, choose the topic you want.

   For the first lesson, you should choose CAR WASH.

2. Click the mouse on CAR WASH.

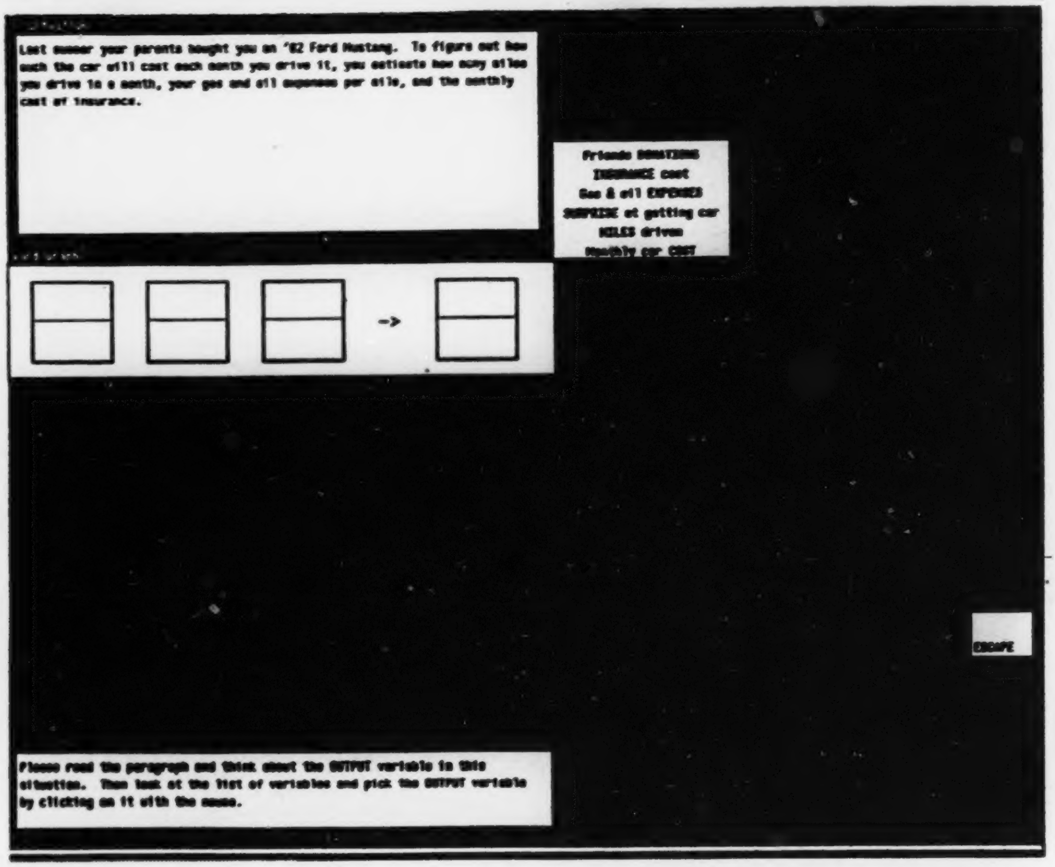

ESC
! 1 @ 2 # 3 $ 4 % 5 ^ 6 & 7 * 8 (9) 0 — – + = ~ `
TAB
Q W E R T Y U I O P { [}]
CTRL
CAPS LOCK
A S D F G H J K L : ; " '
RETURN
SHIFT
Z X C V B N M < , > . ? /
SHIFT

The Keyboard

LOGOUT:

1. If you haven't finished your topic, click the mouse on ESCAPE or press the ESCAPE button.
2. When you see the list of topics, click the mouse on QUIT.

Draw a circle around the instructions ("Specific Problem") window. Always look here to find out what to do.



Last summer your parents bought you an '82 Ford Mustang. To figure out how much the car will cost each month you drive it, you estimate how many miles you drive in a month, your gas and oil expenses per mile, and the monthly cost of insurance.

Please read the paragraph and think about the OUTPUT variable in this situation. Then look at the list of variables and pick the OUTPUT variable by clicking on it with the mouse.

Friends DONATIONS
INSURANCE cost
Gas & oil EXPENSES
SURPRISE at getting car
MILES driven
Monthly car COST
->
ESCAPE

Picking the OUTPUT Variable

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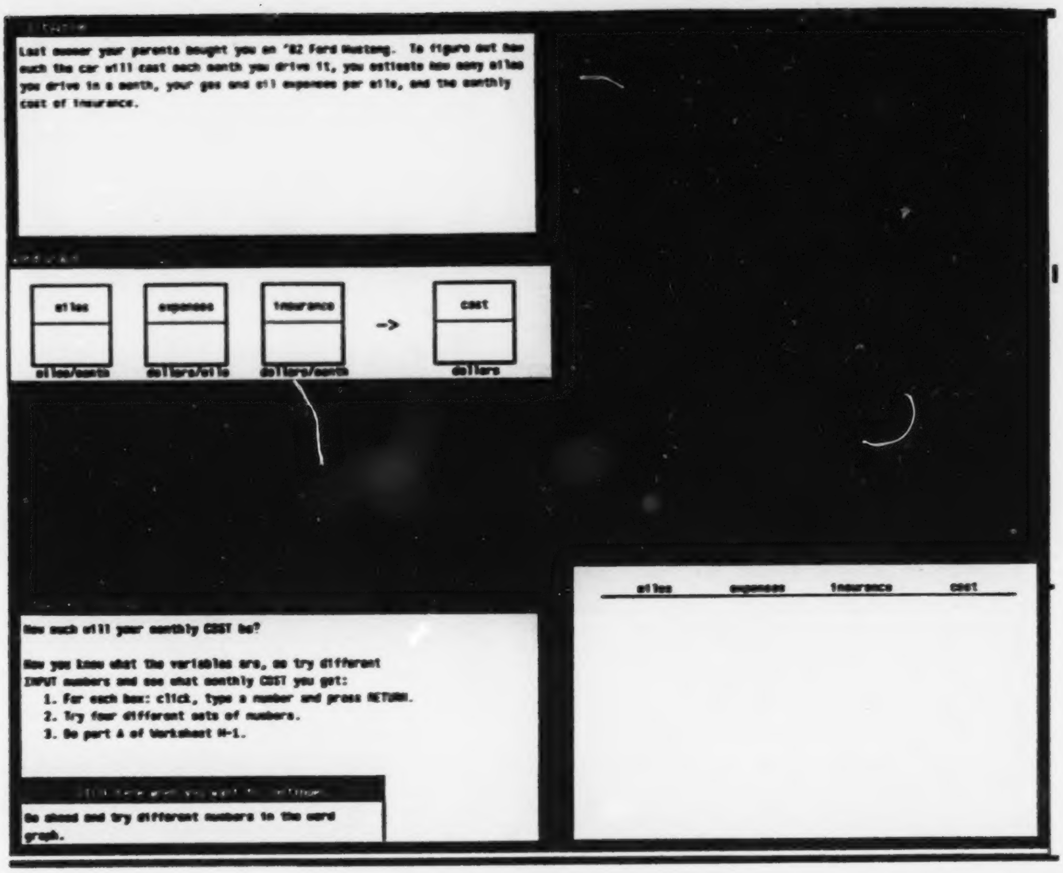

miles
expenses
insurance
->
cost
miles/month
dollars/mile
dollars/month
dollars

Last summer your parents bought you an '82 Ford Mustang. To figure out how much the car will cost each month you drive it, you estimate how many miles you drive in a month, your gas and oil expenses per mile, and the monthly cost of insurance.

How much will your monthly COST be?

Now you know what the variables are, so try different INPUT numbers and see what monthly COST you get:

1. For each box: click, type a number and press RETURN.
2. Try four different sets of numbers.
3. Do part A of Worksheet M-1.

Go ahead and try different numbers in the word graph.

miles	expenses	insurance	cost

Using the WORD GRAPH and TABLE

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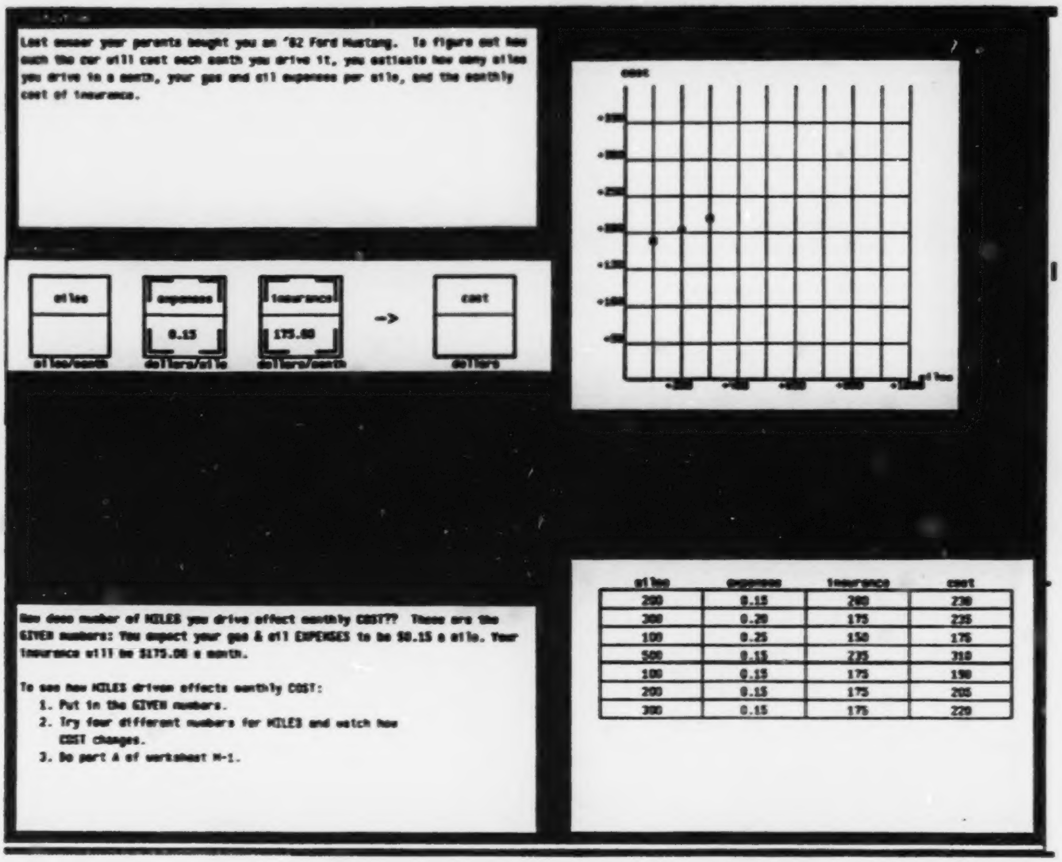

Last summer your parents bought you an '82 Ford Mustang. To figure out how much the car will cost each month you drive it, you estimate how many miles you drive in a month, your gas and oil expenses per mile, and the monthly cost of insurance.

miles: (miles/month)
expenses: 0.15 (dollars/mile)
insurance: 175.00 (dollars/month)
-> cost: (dollars)

How does number of MILES you drive affect monthly COST?? These are the GIVEN numbers: You expect your gas & oil EXPENSES to be $0.15 a mile. Your insurance will be $175.00 a month.

To see how MILES driven affects monthly COST:

1. Put in the GIVEN numbers.
2. Try four different numbers for MILES and watch how COST changes.
3. Do part A of worksheet M-1.

miles	expenses	insurance	cost
200	0.15	200	230
300	0.20	175	235
100	0.25	150	175
500	0.15	235	310
100	0.15	175	190
200	0.15	175	205
300	0.15	175	220

Exploring How One Variable Affects Another

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EXPLORING WORD SITUATIONS -- EXPLORATION A
CAR WASH

NAMES: ______________________ ______________________

DATE: __________ TEACHER: ______________ PERIOD: ____

1. What was the output variable?

2. If you increased the number of cars washed and kept everything else the same, what do you think should happen to the profit?

3. If you increased the amount spent on supplies and kept everything else the same, what do you think should happen to the profit?

4. Copy the line of the table that you thought gave the most realistic profit:

TABLE

CARS	FEE	SUPPLIES	DONATION	PROFIT

****************************** GENERAL NOTES ***********************************

In the space below, please feel free to record:

-- Anything you noticed that was interesting or unusual
-- Questions you might have
-- Things to remember for exploring future situations

EXPLORING WORD SITUATIONS -- EXPLORATION B
CAR WASH

1. Which input variable in the word graph could you change yourself?

2. As you decreased this variable, what happened to your profit?

3. What were the SMALLEST and LARGEST values you got for profit?

SMALLEST: ____________ LARGEST: ____________

What input amounts gave you these profits? List your answers in this chart:

	INPUT AMOUNT	PROFIT
SMALLEST		
LARGEST		

4. What was happening on the graph as you put numbers into the variable box? (Please give your observations. There is no correct answer!)

****************************** GENERAL NOTES ********************************

In the space below, please feel free to record:

-- Anything you noticed that was interesting or unusual
-- Questions you might have
-- Things to remember for exploring future situations

UNDERSTANDING WORD SITUATIONS

For each situation, answer the questions, list the variables and write at least one question that interests you about each situation.

Example: When cooking a turkey, you need to think about the weight of the bird and the temperature to cook it. Turkeys usually cook at 325 degrees. A six-pound turkey needs to cook for 2 hours, while an eight pound turkey needs 2 hours and 40 minutes at the same temperature. Your family plans to serve a twenty pound bird at 4:00.

A. Answer the following questions about the situation:

1. At what temperature do you cook turkey?
   Turkey is cooked at 325 degrees.
2. What do you need to know about the turkey to know its cooking time?
   You need to know how much the turkey weighs and what temperature to cook it at.

B. List the variables:
Turkey weight *Cooking time* *Temperature*

C. Write at least one question of interest based on the situation:
1. *How long does a twenty-pound turkey take to cook?*
2. *What time must the turkey go in the oven to be ready to eat at 4:00 p.m.?*

1. After last year's prom, the Band Parents prepared a pancake breakfast. When calculating the cost of making all the pancakes, they thought about how many couples would come to the breakfast, how many pancakes each couple would eat, and how much the batter would cost for each pancake.

A. Answer the following questions:

1. What did the Band Parents try to calculate?

2. What factors did they have to take into account?

B. List the variables:

C. Write at least one question of interest based on the situation:

2. To feed an older puppy, you must give it around six cups of dog food a day. Sparky is our 10 month-old black Labrador puppy. We buy her dry dog food in bags. Each bag of dog food we buy for her contains twenty-five cups of food.

A. Answer the following questions:

1. How many cups of food should we give Sparky each day?
2. How big are the bags of dog food?

B. List the variables:

C. Write at least one question of interest based on the situation:

WORD SITUATIONS: VARIABLES AND WORD GRAPHS

For each situation, answer the questions, list the variables and write a word graph.

Example: At the end of last summer, your parents bought you an '82 Ford Mustang. To figure out how much the car will cost you each month, you estimate how many miles you drive in a month, and then how much gas and oil cost you per mile. You also add in your insurance. With your clean driving record and good grades, this is not as much as some students have to pay.

A. Answer the following questions about the situation:

1. What kind of car do you have?
   My parents gave me an '82 Ford Mustang.
2. What car expenses must you take care of?
   My car expenses are gas and oil, and insurance.

B. The output variable is "cost of running the car". List the input variables:

Insurance *Miles driven* *Cost per mile*

C. Write a word graph to describe the situation:

Miles driven *Cost per mile* *Insurance* ===> *Cost of running car*

3. Last summer you worked for the Recreation and Parks Department, cleaning the area south of Santa Monica Pier. You enjoyed the work. The people you met on the job were very friendly. The work was somewhat flexible, since the department paid you by the hour, and the number of hours you worked would change from week to week.

A. Answer the following questions about the situation:

1. For whom did you work last summer?

2. How were you paid?

3. What was your work schedule like?

B. The output variable is "weekly pay." List the input variables:

C. Write a word graph to describe this situation:

===> *Weekly pay*

4. At last year's prom, your sister and her friends rented a limousine. To find a limousine that was not too expensive, they called several different companies. They asked about the rate per hour, and decided how many hours to rent the limo. In adding up their total cost, they also remembered they would have to add a tip for the driver.

A. Answer the following questions about the situation:

1. Why did your sister and her friends rent a limo?
2. What did they ask each limousine company?
3. Is the driver's tip included in the hourly rate?

B. What is the output variable?

List the input variables:

C. Write a word graph for the situation:

====>

USING TABLES: INCREASING AND DECREASING RELATIONSHIPS

For each situation, complete the table and answer the questions.

5. Last summer you worked for the Recreation and Parks Department. Based on the word graph you wrote, you can make a word equation to show how much you would earn before deductions are taken out.

Number of hours * Hourly wage = Pay

A. Use this word equation and your calculator to complete the table:

Hours *	Hourly wage =	Pay
16	4.25	
20	4.25	
32	4.25	
45	4.25	
88	4.25	
120	4.25	

B. Answer the following questions about the table:

1. What is your pay for 16 hours of work? 45 hours? 120 hours?
2. How many hours did it take to get paid $85? $136? $374?
3. Does increasing your hours increase or decrease your pay?
4. Does decreasing your hours increase or decrease your pay?
5. What input variable stayed the same in this table?

6. You didn't receive all of your pay because of deductions. You calculate that 85 cents per hour was deducted. So your take home wage was $3.40 per hour. The word equation which shows how much pay you took home looks like this:

Number of hours * Take-home wage = Take-home pay

A. Use the word equation and your calculator to complete the table:

Hours *	Wage	= Take-home pay
16	3.40	
20	3.40	
32	3.40	
45	3.40	
110	3.40	
120	3.40	

B. Answer the following questions about the table:

1. What is your take-home pay for 120 hours? 32 hours? 20 hours?
2. How many hours did you work to take home $374? $153? $68?
3. What effect does increasing your hours have on your take-home pay?

*7. Answer the following questions, based on the tables in 5 and 6:

A. How many hours did you have to work to make $374 before deductions?

How many hours did you work to take home $374?

For which did you have to work more hours, taking home $374 or getting $374 before deductions?

B. How many hours did you have to work to make $68 before deductions?

How many hours did you work to take home 68?

For which did you have to work more hours, taking home $68 or getting $68 before deductions?

What was the difference in hours you had to work?

C. To take home the same amount of money, do your hours have to increase or decrease when deductions are taken out?

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8. Your older sister and her boyfriend are traveling to Europe this fall. They plan to visit England, France and West Germany. Each US dollar is worth .56 pounds, 5.90 francs and 1.75 marks. Their flight lands in Heathrow, where they exchange some traveler's checks for pounds and pay a commission of two pounds. You can write a word graph about this situation:

Number of dollars   Exchange rate   Commission ===> Number of pounds

The word equation is:

(Number of dollars * Exchange rate) - Commission = Number of pounds

A. Use the word equation and your calculator to complete the table:

(Number of dollars *	Exchange rate) -	Commission =	Number of pounds
250	.56	2	
200	.56	2	
300	.56	2	
150	.56	2	
325	.56	2	
160	.56	2	

B. Answer the following questions based on the table:

1. How many pounds do you get if you change $250? $160? $325?
2. How many dollars would you change to get 82 pounds?

   110 pounds?   166 pounds?
3. What effect does increasing the number of dollars have on the number of pounds you get?
4. What effect does increasing the number of dollars have on the commission you pay?

9. Their second stop is southern France, which reminds them of Santa Monica. When they go to get French francs, they must pay a commission of 20 francs. The word equation for francs is:

(Number of dollars * Exchange rate) - Commission = Number of francs

A. Use the word equation and your calculator to complete the table:

(Number of dollars *	Exchange rate) -	Commission =	Number of francs
250	5.90	20	
200	5.90	20	
300	5.90	20	
150	5.90	20	
325	5.90	20	
160	5.90	20	

B. Answer the following questions based on the table:

1. What effect does decreasing the number of dollars have on the number of francs you get?
2. What effect does increasing the number of dollars have on the commission you must pay?
3. What effect does increasing the number of dollars have on the exchange rate?

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HANDOUT FOR WORKSHEET M2

5. PAY

A. Complete the table by calculating the pay.

Hours *	Hourly wage =	Pay
16	4.25	
20	4.25	
32	4.25	
45	4.25	
88	4.25	
120	4.25	

B. Answers:

1. 16 hours: ________ 45 hours: ________ 120 hours ________

2. $85: __________ $136: __________ $374: __________

3. ________________

4. ________________

5. ________________

6. TAKE-HOME PAY

A. Complete the table by calculating the take-home pay.

Hours *	Wage =	Take-home pay
16	3.40	
20	3.40	
32	3.40	
45	3.40	
110	3.40	
120	3.40	

B. Answers:

1. 120 hours: ________ 32 hours: ________ 20 hours: ________

2. $374: ____________ $153: ____________ $68:______________

3. ____________________

8. EXCHANGING DOLLARS FOR POUNDS

A. Complete the table:

(Number of dollars *	Exchange rate) -	Commission =	Number of pounds
250	.56	2	
200	.56	2	
300	.56	2	
150	.56	2	
325	.56	2	
160	.56	2	

B. Answers:

1. $250: ____________ $160: ____________ $325: ____________

2. 82 pounds: ________ 110 pounds: ________ 166 pounds: ________

3. ____________________

4. ____________________

9. EXCHANGING DOLLARS FOR FRANCS

A. Complete the table:

(Number of dollars *	Exchange rate) -	Commission =	Number of francs
250	5.90	20	
200	5.90	20	
300	5.90	20	
150	5.90	20	
325	5.90	20	
160	5.90	20	

B. Answers:

1. ______________________

2. ______________________

3. ______________________

GROUP EXERCISE: JIMMY Z T-SHIRT SALE

Surf-And-Sun clothing store has just opened at the revitalized mall. The sales manager is planning a special sale of Jimmy Z T-shirts to attract business. To find out how much the special sale will cost the store, the manager must take into account all these factors: the number of Jimmy Z T-shirts the store will order for the sale, how much Jimmy Z will charge for each shirt, money for advertising, and the amount for hiring a temporary salesperson. Fortunately, the manager can also count on a "new business" rebate Jimmy Z has promised to give the store.**

In your group, discuss the variables in this situation.

1. What is the output variable?

2. What are the input variables?

A. D.

B. E.

C.

3. Write a word graph which describes the situation:

______ ______ ______ ______ ______ ====> ______

** A REBATE is money that a seller returns to the buyer. For example, a person who buys a new car often pays the full price, but then gets money back from the dealer or the car manufacturer (like Ford or GM). This is called a rebate.

4. For each input variable, answer the following questions. Write your answers on the chart below.

- -- Write the name of the variable in the first row.
- -- Write the effect of the variable in the second row: Does the variable make the output increase or decrease?
- -- Write the SIZE of the effect in the third row: If you change the variable by 1, will the output also change by 1, or will it change by a bigger number?
- -- If the variable makes a "big" change in the output, answer this question: What other input variable is it multiplied with? Put your answer in row four.
- -- In the last row, write the operator which should go with the variable: + , - , or *. (Hint: You should have two +'s, one -, and two variables multiplied together.)

	Variable A	Variable B	Variable C	Variable D	Variable E
Name of Variable					
Effect: Increase or decrease					
Size of Effect: 1 or "big"					
Multiplied by...					
Operator: + , - , *					

5. Based on the chart, write the word equation which describes the situation:

(______ * ______) + ______ + ______ - ______ = ______

SOLVING PROBLEMS WITH TABLES AND GRAPHS
EXPLORATION A

NAMES: ________________________ ________________________

DATE: __________ TEACHER: ______________ PERIOD: ____

TOPIC NAME: ________________

1. What question were you trying to answer?

2. What answer did you get? (Answer in a short sentence.)

3. In the list below, circle the way(s) you found your answer: (You can circle more than one.)

    1. Guess and test
    2. Made a lucky guess
    3. Thought about the situation and estimated the amount
    4. Tried numbers until I got the right one
    5. Did some math to get the amount
    6. Figured out the answer in my head, with pencil & paper, or calculator
    7. Looked on graph
    8. Looked in table

4. Suppose your partner were absent. How would you explain to your partner how you got your answer?

***************************** GENERAL NOTES ***********************************

In the space below, please feel free to record:

-- Anything you noticed that was interesting or unusual
-- Questions you might have
-- Things to remember for exploring future situations

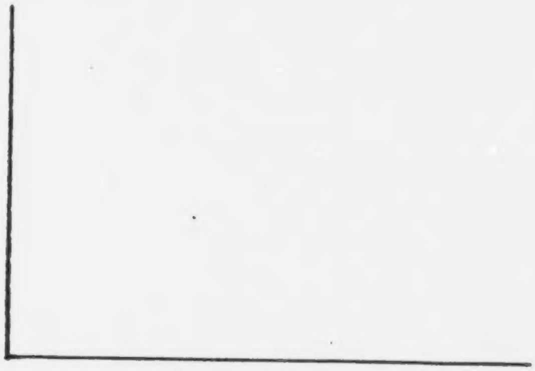
SOLVING PROBLEMS -- EXPLORATION B

1. What question were you trying to answer?

2. What answer did you get? (Answer in a short sentence.)

3. For a record of this exploration, label the axes with the names of the variables. Then sketch the diagonal line from the graph:

OUTPUT: ________



INPUT: ________

4. Suppose you wanted the output variable to come out very HIGH. Using the graph, figure out approximately what the amount of the input variable should be. Put your answer in this table:

HIGH OUTPUT VARIABLE AMOUNT	ESTIMATED INPUT VARIABLE AMOUNT

5. Suppose your partner were absent. How would you explain to your partner how you used the graph to get your answer?

ANSWERING QUESTIONS USING TABLES AND GRAPHS

Read the paragraph about blood pressure and look at the table and graph. Then go to page 2 and solve the specific problems.

Blood pressure depends upon many factors -- age, weight, health. When the Kaiser Mobile Health Center arrived at Samohi, teachers and staff volunteered to have their blood pressure taken. The Kaiser health workers had a formula for finding what a person's average blood pressure should be based on their age.

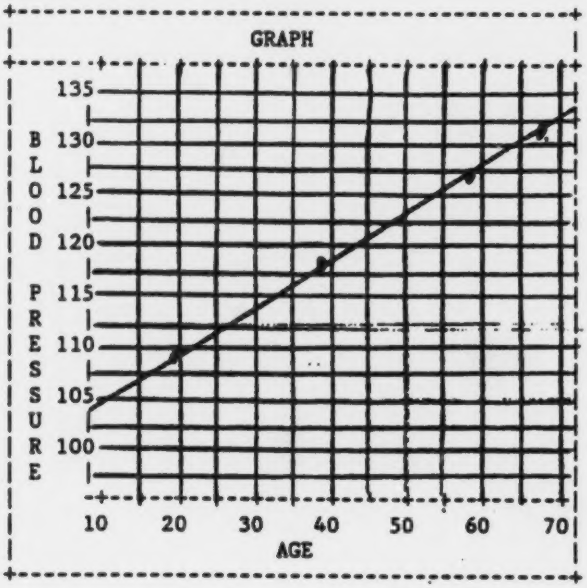
Written as a word equation, the formula relating blood pressure and age is:

(Age of person * .44) + 101 = Average blood pressure

The word equation has been used to make a table. The numbers in the table are also shown in a graph of BLOOD PRESSURE against AGE.

TABLE

(Age	* .44)	+ 101	= Blood pressure
38	.44	101	118
19	.44	101	109
68	.44	101	131
50	.44	101	123
58	.44	101	127



Write your answer to each question on the handout sheet.

Problem 1: A 38 year-old woman wants to know what the average blood pressure for her age is.

A. Using a color pencil, put a box around the TARGET VARIABLE in the TABLE
on your handout.

B. Find 38 in the AGE column of the table: What should the blood-pressure of a 38-year old woman be? (Answer with a complete sentence.)

Follow-up questions: Use the table to answer.

C. What should the blood pressure of a 58-year old be?

D. The blood pressure of a 19-year old?

Problem 2: About what blood pressure does the average 40-year old have?

A. Using a color pencil, put a box around the TARGET VARIABLE in the GRAPH on your handout.

B. Find 40 years on the part of the graph that says "AGE." Draw a line straight upwards from the 40 until you reach the slanted line. From this point, draw a line to the left until you reach the part of the graph that says "BLOOD PRESSURE." What number do you hit there?

C. Explain your answer in a complete sentence.

Follow-up questions: Draw lines on the graph to estimate your answers.

D. What blood pressure should a 65-year old have?

E. A 25-year old?

F. How old are you?

According to the graph, what should your own blood pressure be?

If you have had your blood pressure measured before, what was it?

How different is it from what you estimated on the graph?

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Problem 3: One of the teachers finds that his blood pressure is 127. If this teacher has an average blood pressure, what is his age?

A. Using a black pencil, draw a box around the TARGET VARIABLE in the TABLE on your handout.

B. Find 127 in the table under "BLOOD PRESSURE": 127 is the usual blood pressure for what age? (Explain in a complete sentence.)

Follow-up questions: Use the table to answer.

C. What is the usual age for a person with blood pressure of 131?

D. With blood pressure of 109?

E. With blood pressure of 123?

Problem 4: What is the usual age of a person with blood pressure of 115?

A. Using a black pencil, draw a box around the TARGET VARIABLE in the GRAPH on your handout.

B. Find 115 on the part of the graph that says "BLOOD PRESSURE". Draw a line to the right until you reach the slanted line. Then draw straight down to the part of the graph that says "AGE." What number do you hit?

C. Explain your answer to the problem in a complete sentence.

Follow-up questions: Draw lines on the graph to estimate your answers.

D. What is the usual age of a person with blood pressure of 112?

E. With blood pressure of 135?

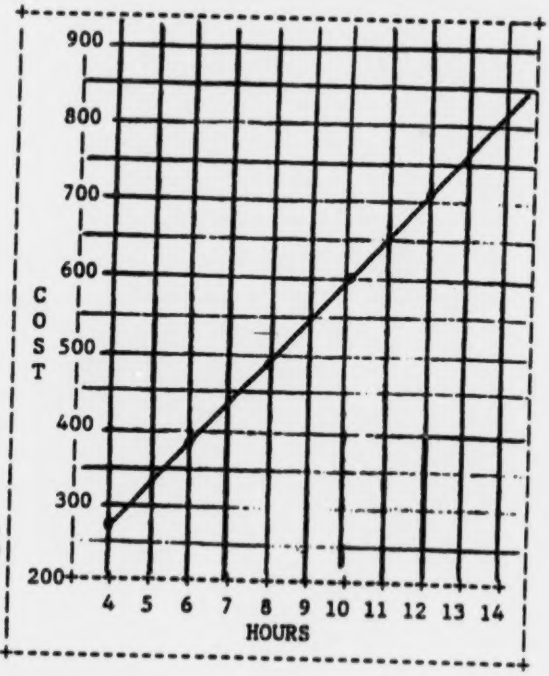
Read the paragraph about renting a limousine. Then solve problems 5 and 6.

> When your sister and her friends rented a limousine for last year's prom, they called several companies to ask about hourly rates. They thought about how many hours to rent the limo, and remembered to add a tip for the driver. The word equation for this situation might be:
>
> (Hours * Hourly rate) + Tip = Total cost

Problem 5: AAA Limo has the perfect limousine at $55 per hour, with a $50 tip. If your sister's friends want to keep the limo for the prom and the prom breakfast (8 pm Friday to 6 am Saturday), how much will they spend?

A. What is the target variable in problem 5? (Box it on the table and graph.)

Hours	* Rate	+ Tip	= Cost
10	55	50	600
6	55	50	380
12	55	50	710
4	55	50	270
8	55	50	490



B. Use the table to solve the problem: How much will it cost to use the limo from 8 pm to 6 am?

Follow-up questions:

C. If they use the limo just to go to the prom (8 pm to midnight), how much will it cost? (Use table.)

D. If they go to the prom and after-prom party (8 pm to 4 am), what will the total cost be?

Use the graph to answer the following questions:

E. Approximately how much would it cost to rent the limo for 5 hours?

F. For 7 hours?

G. For 14 hours?

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Problem 6: Your sister and her friends decide to keep the limo for the prom, after-prom party and breakfast, a total of 10 hours. They decide they want to spend no more than $800. Their tip will still be $50, but they call other companies to compare their hourly rates with AAA's rate. Which companies will cost less than $800?

A. What is the target variable in problem 6? (Box it on the table and graph.)

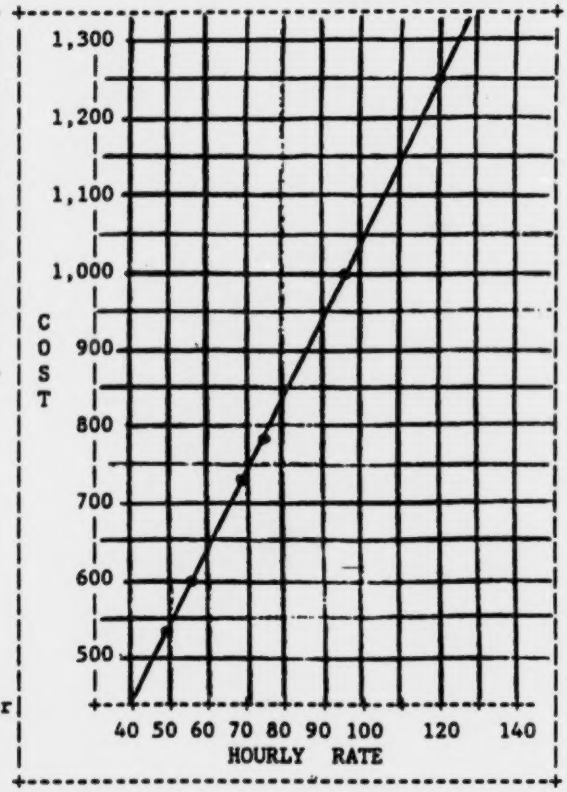
	Hours	* Rate	+ Tip	= Cost
AAA	10	55	50	600
Joe's	10	73	50	780
Crown	10	95	50	1,000
Star	10	68	50	730
Luxury	10	120	50	1,250
Fred's	10	49	50	540

B. Use the table to solve the problem: What companies are within their budget of $800?

Follow-up questions:

C. Use the table to answer: Which companies can they NOT afford?

Use the graph to answer:
D. Your sister also called Silver Moon Limo. Silver Moon's limo would cost them $900. Estimate Silver Moon's hourly rate.



E. If the most they can spend is $1000 instead of $800, estimate what hourly rate they can afford. (Use the graph to answer.)

F. What if the most they can spend is $1,100? (Use the graph to answer.)

*G. Your sister really likes the Crown limousine. What two things could she and her friends change in order to afford Crown's service?

PRACTICE USING SIGNED NUMBERS

What did the BOY VOLCANO say to the GIRL VOLCANO?

TO DISCOVER THE ROMANTIC WORDS OF THE BOY VOLCANO:

Do any exercise below and find your answer at the bottom of the page. Shade in the letter above each correct answer.

WHEN YOU FINISH, THE BOY VOLCANO'S WORDS WILL REMAIN!

① $^{-}178 + {}^{-}345 =$

② $^{-}403 - {}^{-}177 =$

③ $({}^{-}397)(8) =$

④ $\frac{^{-}2632}{^{-}7} =$

⑤ $690 + {}^{-}255 =$

⑥ $800 - {}^{-}499 =$

⑦ $({}^{-}56)({}^{-}90) =$

⑧ $\frac{^{-}7080}{10} =$

⑨ $^{-}83 + 24 + {}^{-}19 =$

⑩ $^{-}2094 - 67 =$

⑪ $({}^{-}12)({}^{-}12)({}^{-}12) =$

⑫ $\frac{4956}{^{-}84} =$

⑬ $^{-}15 + 19 + {}^{-}26 + 11 =$

⑭ $^{-}232 - {}^{-}508 =$

⑮ $({}^{-}5)(6)({}^{-}7)(8) =$

⑯ $\frac{0}{^{-}600} =$

⑰ $76 + {}^{-}90 - {}^{-}48 =$

⑱ $\left(\frac{^{-}3768}{^{-}6}\right)({}^{-}3) =$

E	V	E	R	Y	S	M	I	L	E	A	D	D	S	F	A	C	E	V	A	L	U	E	!
1299	276	⁻523	⁻1884	⁻3176	1680	5040	45	⁻613	⁻2161	⁻11	⁻226	435	0	⁻59	2480	376	⁻78	285	⁻1974	⁻708	⁻2976	34	⁻1728

Why did the ant run across the cracker box?

Do any exercise below and find your answer in the corresponding answer column. The letter of the exercise goes in the box that contains the number of the answer. Keep working and you will discover the answer to the title question.

$\boxed{T}\ {}^{-}15 + 7 =$	㉓ 7
$\boxed{A}\ 8 - {}^{-}21 =$	㉜ ${}^{-}27$
$\boxed{D}\ (3)({}^{-}9) =$	㉘ 50
$\boxed{H}\ {}^{-}24 \div 4 =$	⑬ ${}^{-}8$
$\boxed{E}\ {}^{-}9 + {}^{-}13 =$	㉕ ${}^{-}6$
$\boxed{O}\ ({}^{-}2)({}^{-}25) =$	㊱ ${}^{-}5$
$\boxed{L}\ {}^{-}50 - 30 =$	⑤ 29
$\boxed{G}\ {}^{-}56 \div {}^{-}8 =$	⑧ ${}^{-}80$
$\boxed{E}\ 32 + {}^{-}37 =$	③ ${}^{-}22$

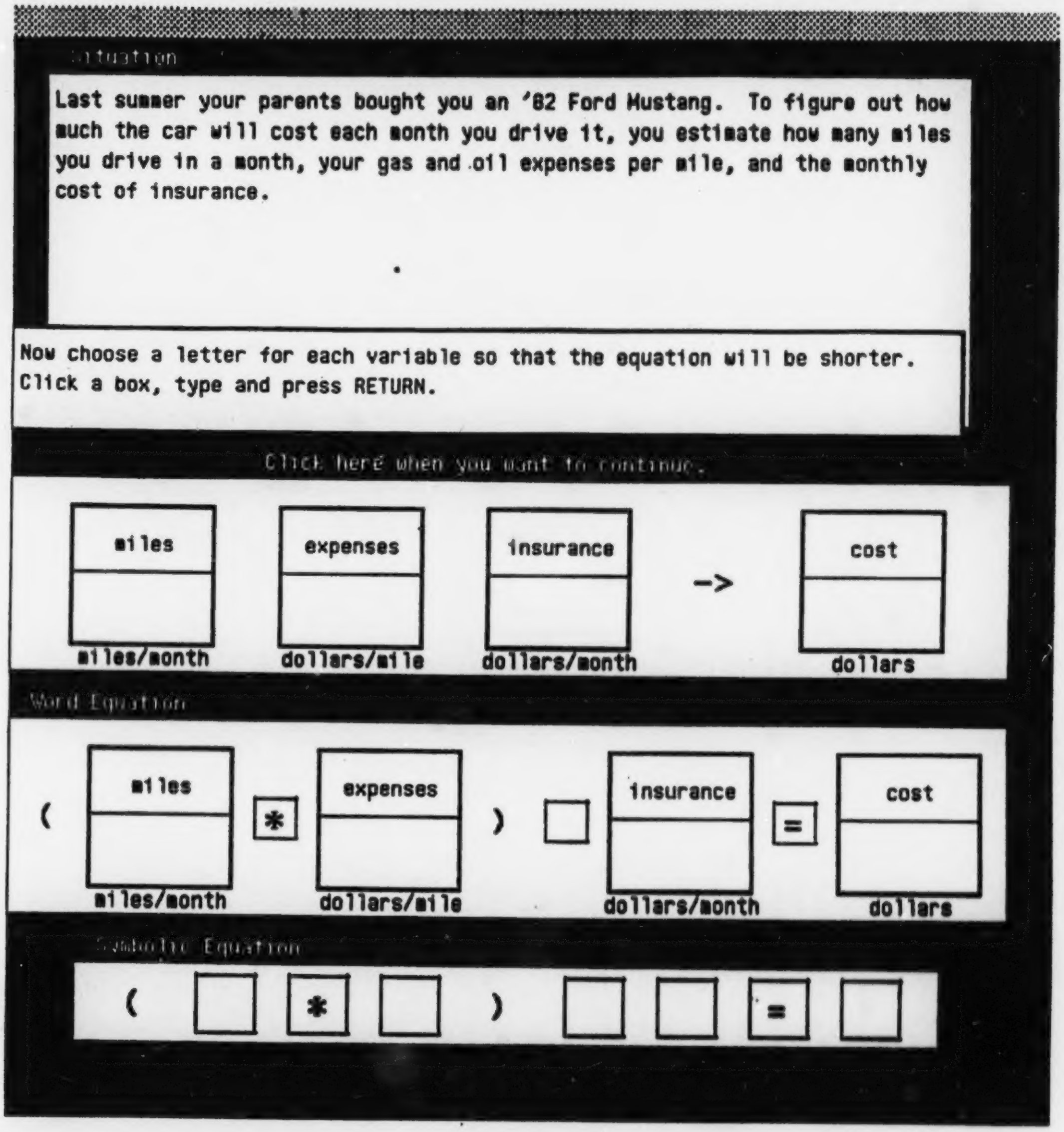
$\boxed{T}\ ({}^{-}15 \div 3) + 14 =$	② ${}^{-}1$
$\boxed{E}\ ({}^{-}10 + {}^{-}5)({}^{-}2) =$	④ 3
$\boxed{H}\ ({}^{-}3 - 4) \div 7 =$	⑩ ${}^{-}55$
$\boxed{D}\ ({}^{-}9 \cdot 6) + {}^{-}4 =$	⑦ 30
$\boxed{O}\ ({}^{-}30 - {}^{-}22) \cdot 6 =$	⑫ ${}^{-}58$
$\boxed{A}\ (20 \div 4) \cdot {}^{-}11 =$	㉞ 100
$\boxed{E}\ (28 - {}^{-}10) - 7 =$	㉚ 9
$\boxed{I}\ ({}^{-}13 + {}^{-}12)({}^{-}4) =$	㉛ 31
$\boxed{L}\ (4 \cdot {}^{-}6) \div {}^{-}8 =$	㉑ ${}^{-}48$

$\boxed{I}\ {}^{-}5 \cdot 20 =$	⑨ 12
$\boxed{T}\ 30 \div {}^{-}2 =$	⑪ ${}^{-}100$
$\boxed{A}\ {}^{-}9 - {}^{-}19 =$	㉒ 77
$\boxed{N}\ {}^{-}7 \cdot {}^{-}11 =$	㉖ ${}^{-}14$
$\boxed{O}\ {}^{-}7 + {}^{-}11 =$	㉔ ${}^{-}15$
$\boxed{S}\ {}^{-}60 \div {}^{-}5 =$	① ${}^{-}24$
$\boxed{T}\ 12 - 36 =$	㉝ ${}^{-}26$
$\boxed{E}\ {}^{-}17 - {}^{-}3 =$	⑭ ${}^{-}18$
$\boxed{L}\ \frac{260}{{}^{-}10} =$	⑰ 10

$\boxed{E}\ ({}^{-}6 + 17) - 20 =$	⑱ 14
$\boxed{A}\ ({}^{-}64 \div 2) \div {}^{-}2 =$	⑳ 2
$\boxed{B}\ ({}^{-}5 - {}^{-}6) \cdot {}^{-}87 =$	⑯ ${}^{-}9$
$\boxed{T}\ ({}^{-}40 + {}^{-}50) \div 9 =$	㉟ ${}^{-}12$
$\boxed{R}\ ({}^{-}13 \cdot {}^{-}2) + {}^{-}12 =$	⑥ ${}^{-}87$
$\boxed{N}\ (42 \div {}^{-}7) - 6 =$	㉗ 75
$\boxed{D}\ ({}^{-}5 - {}^{-}30)(3) =$	⑮ 72
$\boxed{L}\ ({}^{-}12 + {}^{-}18) \div {}^{-}15 =$	㉙ ${}^{-}10$
$\boxed{T}\ ({}^{-}8 \cdot {}^{-}8) - {}^{-}8 =$	⑲ 16

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36

INTRODUCTION TO EQUATIONS



Situation

Last summer your parents bought you an '82 Ford Mustang. To figure out how much the car will cost each month you drive it, you estimate how many miles you drive in a month, your gas and oil expenses per mile, and the monthly cost of insurance.

Now choose a letter for each variable so that the equation will be shorter. Click a box, type and press RETURN.

Click here when you want to continue.
miles
miles/month
expenses
dollars/mile
insurance
dollars/month
->
cost
dollars
Word Equation
(miles * expenses) insurance = cost
miles/month
dollars/mile
dollars/month
dollars
Symbolic Equation
(*) =

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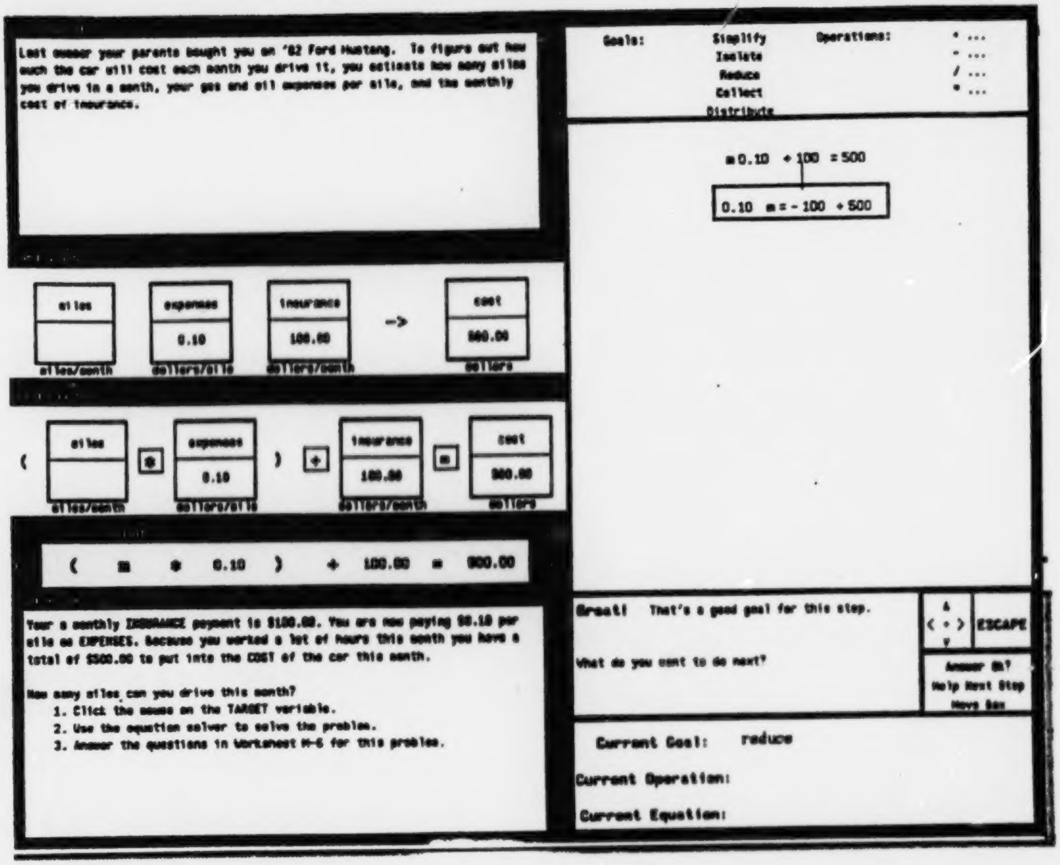
INTRODUCTION TO EQUATIONS



Last summer your parents bought you an '82 Ford Mustang. To figure out how much the car will cost each month you drive it, you estimate how many miles you drive in a month, your gas and oil expenses per mile, and the monthly cost of insurance.

miles
miles/month
expenses
0.10
dollars/mile
insurance
100.00
dollars/month
->
cost
500.00
dollars
(
miles
miles/month
*
expenses
0.10
dollars/mile
)
+
insurance
100.00
dollars/month
=
cost
500.00
dollars

(m * 0.10) + 100.00 = 500.00

Your a monthly INSURANCE payment is $100.00. You are now paying $0.10 per mile on EXPENSES. Because you worked a lot of hours this month you have a total of $500.00 to put into the COST of the car this month.

How many miles can you drive this month?

1. Click the mouse on the TARGET variable.
2. Use the equation solver to solve the problem.
3. Answer the questions in Worksheet M-6 for this problem.

Goals:
Simplify
Isolate
Reduce
Collect
Distribute
Operations:
+ ...
- ...
/ ...
* ...

m 0.10 + 100 = 500

0.10 m = - 100 + 500

Great! That's a good goal for this step.

What do you want to do next?

ESCAPE
Answer Ok?
Help Next Step
Move Box

Current Goal: reduce

Current Operation:

Current Equation:

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SOLVING PROBLEMS WITH EQUATIONS

NAMES: ______________________ ______________________

DATE: __________ TEACHER: ____________ PERIOD: ____

*** PLEASE DO AS MANY OF THE PROBLEMS AS YOU HAVE TIME FOR ***

1. CAR WASH, Problem 1

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

2. CAR WASH, Problem 2

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

3. HEAL THE BAY-I, Problem 1

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

4. HEAL THE BAY-I, Problem 2

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

5. HEAL THE BAY-II, Problem 1

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

6. HEAL THE BAY-II, Problem 2

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

7. FISHING TRIP, Problem 1

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

8. FISHING TRIP, Problem 2

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

SOLVING EQUATIONS: SIMPLIFY AND REDUCE

Read the paragraph about changing money. Then solve problems 1 and 2.

Last summer you and four friends went to Baja. All the money you spent on meals, lodging and gas was in pesos. You converted your U.S. dollars into pesos according to this word equation:

Number of DOLLARS * Exchange RATE = Number of PESOS

You could write a symbolic equation like this:

D * R = P

Problem 1: The exchange rate is 2,174 pesos for each U.S. dollar. How many pesos did you spend on meals, lodging and gas, if you changed $375?

A. What is the TARGET VARIABLE in problem 1?

B. Putting the numbers into the symbolic equation, you have:

375 * 2,174 = P

Copy the equation.

C. What is your first GOAL in solving the equation?

D. Carry out the GOAL and solve the problem.

E. How many pesos did you spend? (Answer with a complete sentence.)

Problem 2: The exchange rate was 2,174 pesos to a U.S. dollar. You wanted to exchange enough dollars to get 543,500 pesos.

A. What is the TARGET VARIABLE in problem 2?

B. Putting the numbers into the symbolic equation, you have:

D * 2,174 = 543,500

Copy the equation.

C. What is your first GOAL in solving the equation?

3-12. Copy the equation and write the GOAL that would be the first step in solving the equation: either SIMPLIFY or REDUCE. You do not have to solve. Just list the first GOAL.

3. 77 - 19 = S 1ST GOAL: ____________________

4. P = 336 - 500 1ST GOAL: ____________________

5. 62 = 2Y 1ST GOAL: ____________________

6. 5.67W = 89.32 1ST GOAL: ____________________

7. 78 - 47 = T 1ST GOAL: ____________________

8. 480 = C * 24 1ST GOAL: ____________________

9. 199 + 34 = R 1ST GOAL: ____________________

10. W = 46 * 71 1ST GOAL: ____________________

11. 1.08 - 5.92 = S 1ST GOAL: ____________________

12. 56N = 25 + 97 1ST GOAL: ____________________

SOLVING EQUATIONS: PRACTICE USING SIMPLIFY AND REDUCE

1-7. SIMPLIFY each equation:

1. $X = 105 - 35$

2. $-17 - 82 = S$

3. $Y = -43 + 64 - 52$

4. $2.3 + 1.2 = R$

5. $W = \frac{2}{3} + \frac{3}{4}$

6. $23(12) = C$

7. $\frac{102}{51} = P$

8-12. For each equation, the first GOAL is to REDUCE. Give the operation needed to carry out the GOAL. Be sure to include the number you would use in the operation.

Example:

35X = 105

GOAL: REDUCE

OPERATION: / 35

8. 45*B = 61

GOAL: REDUCE

OPERATION: ____________

9. -567 = 2K

GOAL: REDUCE

OPERATION: ____________

10. 6C = 30

GOAL: REDUCE

OPERATION: ____________

11. 25 = -6Z

GOAL: REDUCE

OPERATION: ____________

12. 4.50 = 2.30H

GOAL: REDUCE

OPERATION: ____________

13-17. Solve each equation by using REDUCE and SIMPLIFY.

Example: $5W = 25$

GOAL: REDUCE $\frac{5W}{5} = \frac{25}{5}$

GOAL: SIMPLIFY $W = 5$

13. $21 = 3S$

GOAL: REDUCE

GOAL: SIMPLIFY

14. $13T = 40$

GOAL: REDUCE

GOAL: SIMPLIFY

15. $3.4 = 5C$

GOAL: REDUCE

GOAL: SIMPLIFY

16. $121 = 36D$

GOAL: REDUCE

GOAL: SIMPLIFY

17. $57R = 19$

GOAL: REDUCE

GOAL: SIMPLIFY

18-24. Solve the equation: For each step, give the GOAL (either SIMPLIFY or REDUCE), then do the math. Be sure to CHECK your answer.

18. $439Y = 612$

19. $1.50 = -.25A$

20. $5X = 12 + 3$

21. $49 - 7 = 6W$

22. $3T = 3.6$

23. $-25C = 45 + 13$

24. $.35 + .15 = .20F$

SOLVING PROBLEMS WITH EQUATIONS

NAMES: ____________________ ____________________

DATE: __________ TEACHER: ____________ PERIOD: ___

*** PLEASE DO AS MANY OF THE PROBLEMS AS YOU HAVE TIME FOR ***

1. CAR WASH, Problem 1

What was the TARGET VARIABLE? ____________________
Explain your answer in a short sentence:

2. CAR WASH, Problem 2

What was the TARGET VARIABLE? ____________________
Explain your answer in a short sentence:

3. PAY CHECK, Problem 1

What was the TARGET VARIABLE? ____________________
Explain your answer in a short sentence:

4. PAY CHECK, Problem 2

What was the TARGET VARIABLE? ____________________
Explain your answer in a short sentence:

5. PIZZA HEAVEN, Problem 1

What was the TARGET VARIABLE? ____________________
Explain your answer in a short sentence:

6. PIZZA HEAVEN, Problem 2

What was the TARGET VARIABLE? ____________________
Explain your answer in a short sentence:

7. HEAL THE BAY-I, Problem 1

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

8. HEAL THE BAY-I, Problem 2

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

9. HEAL THE BAY-II, Problem 1

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

10. HEAL THE BAY-II, Problem 2

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

11. T-SHIRTS, Problem 1

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

12. T-SHIRTS, Problem 2

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

13. FISHING TRIP, Problem 1

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

14. FISHING TRIP, Problem 2

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

SOLVING EQUATIONS: SIMPLIFY, REDUCE AND ISOLATE

Read the paragraph about changing money. Then solve problems 1 and 2.

> Our travelers to Baja arrived on a Sunday without any pesos. They had to exchange their money at a money changer rather than at the bank. This cost them a hefty exchange fee. The word equation for the situation is:
>
> Number of DOLLARS * Exchange RATE - FEE = Number of PESOS

Problem 1: If the exchange rate was 2,174 pesos to the dollar, and the exchange fee was 10,000 pesos, how many pesos could you get for $400?

A. What symbolic equation can you make from the word equation?

B. What is the TARGET VARIABLE in problem 1?

C. Substituting 400 for number of dollars, 2,174 for exchange rate and 10,000 for the fee, the equation you must solve is:

$$400 * 2174 - 10,000 = P$$

Copy the equation.

D. What is your first GOAL in solving the equation?

E. Carry out the GOAL and complete the problem.

F. How many pesos could you get for $400? (Answer with a complete sentence.)

Problem 2: The rate is 2,174 pesos to the dollar, the fee is 10,000 pesos, and you got a total of 750,900 pesos. How many dollars did you change?

A. What is the TARGET VARIABLE in problem 2?

B. Substitute 2,174 for rate, 10,000 for fee and 750,900 for number of pesos, and write the equation you must solve.

C. What is your first GOAL in solving the equation for problem B?

3-12. Copy the equation and write the GOAL that would be the first step in solving the equation: SIMPLIFY, REDUCE or ISOLATE. You do not have to solve. Just list the first GOAL.

3. 5 + Y = 25 1ST GOAL: ________________

4. 23 = 2 + 3K 1ST GOAL: ________________

5. C - 5 = 16 1ST GOAL: ________________

6. R = 720 * 9 - 14 1ST GOAL: ________________

7. 2X + 199 = 22 1ST GOAL: ________________

8. 2S + 7.2 = 8.8 1ST GOAL: ________________

9. F - 42 = 105 1ST GOAL: ________________

10. 14,000W - 350 = 49,350 1ST GOAL: ________________

11. 4.2L - 3.4 = 6.8 1ST GOAL: ________________

12. 5.16P = 3.95 1ST GOAL: ________________

SOLVING PROBLEMS WITH EQUATIONS

NAMES: ______________________ ______________________

DATE: __________ TEACHER: _____________ PERIOD: ____

*** PLEASE DO AS MANY OF THE PROBLEMS AS YOU HAVE TIME FOR ***

1. CAR WASH, Problem 1

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

2. CAR WASH, Problem 2

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

3. PAY CHECK, Problem 1

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

4. PAY CHECK, Problem 2

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

5. PIZZA HEAVEN, Problem 1

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

6. PIZZA HEAVEN, Problem 2

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

7. HEAL THE BAY-I, Problem 1

What was the TARGET VARIABLE? ______________________________
Explain your answer in a short sentence:

8. HEAL THE BAY-I, Problem 2

What was the TARGET VARIABLE? ______________________________
Explain your answer in a short sentence:

9. HEAL THE BAY-II, Problem 1

What was the TARGET VARIABLE? ______________________________
Explain your answer in a short sentence:

10. HEAL THE BAY-II, Problem 2

What was the TARGET VARIABLE? ______________________________
Explain your answer in a short sentence:

11. T-SHIRTS, Problem 1

What was the TARGET VARIABLE? ______________________________
Explain your answer in a short sentence:

12. T-SHIRTS, Problem 2

What was the TARGET VARIABLE? ______________________________
Explain your answer in a short sentence:

13. FISHING TRIP, Problem 1

What was the TARGET VARIABLE? ______________________________
Explain your answer in a short sentence:

14. FISHING TRIP, Problem 2

What was the TARGET VARIABLE? ______________________________
Explain your answer in a short sentence:

SOLVING EQUATIONS: PRACTICE USING SIMPLIFY, REDUCE AND ISOLATE

Read the paragraph about summer work. Then solve problems 1 and 2.

> Last summer when you worked for Recreation and Parks, you were paid hourly and worked a different number of hours each week. You worked in the chilly early morning fog and in the blazing noon sun. You made good money, even after deductions. This word equation describes the situation:
>
> HOURS * Hourly RATE - DEDUCTIONS = WAGES

Problem 1: The first week, you worked 35 hours at 3.50 an hour, and had 12.50 in deductions for gloves and a Recreation and Parks T-shirt. How much money did you make?

A. What symbolic equation can you make from the word equation?

B. What is the TARGET VARIABLE in problem 1?

C. Substitute 35 for hours, 3.50 for the hourly rate and 12.50 for deductions, and write the equation you must solve.

D. What is the first GOAL in solving the equation?

E. Carry out the GOAL and solve the problem.

F. How much money did you make during your first week of work? (Answer with a complete sentence.)

Problem 2: The last week you worked, you had gotten a raise and were making 4.25 an hour. Your deductions were only 5.25, and your total wages came to $92.50. How many hours did you get paid for?

A. What is the TARGET VARIABLE in problem 2?

B. Substitute 4.25 for the hourly rate, 5.25 for deductions and 92.50 for wages, and write the equation you must solve.

C. What is your first GOAL in solving the equation?

D. Carry out the GOAL and finish solving the equation.

E. How many hours did you work your last week? (Answer with a complete sentence.)

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3-11. For each equation, the first GOAL is to ISOLATE. Give the operation needed to carry out the GOAL. Be sure to include the number you would use in the operation.

Example:

S + 5 = 17

GOAL: ISOLATE

OPERATION: - 5

3. 32 = T - 2

GOAL: ISOLATE

OPERATION: ______________

4. 14 = 2X + 4

GOAL: ISOLATE

OPERATION: ______________

5. -Y - 13 = 3

GOAL: ISOLATE

OPERATION: ______________

6. 64 = 2H - 40

GOAL: ISOLATE

OPERATION: ______________

7. 19Z - 23 = -49

GOAL: ISOLATE

OPERATION: ______________

8. 1,254D + 5,400 = 1,320

GOAL: ISOLATE

OPERATION: ______________

9. 4 - W = 320

GOAL: ISOLATE

OPERATION: ______________

10. 27 = -2 - P

GOAL: ISOLATE

OPERATION: ______________

11. 3.54 + 6.78X = 9.99

GOAL: ISOLATE

OPERATION: ______________

12-13. The following equations have already been solved. You need to write the name of the TARGET VARIABLE and the GOAL used in each step of the solution.

Example:

TARGET VARIABLE: WAGE (W) $W = 4.5(40) - 10.50$

GOAL: SIMPLIFY

$W = 169.50$

12. TARGET VARIABLE: ____ $100 = 4.75H - 18.75$

GOAL: __________

$100 + 18.75 = 4.75H - 18.75 + 18.75$

GOAL: __________

$118.75 = 4.75H$

GOAL: __________

$$\frac{118.75}{4.75} = \frac{4.75H}{4.75}$$

GOAL: __________

$25 = H$

13. TARGET VARIABLE: ____ $97.50 = 3.25(32) - D$

GOAL: __________

$97.50 = 104 - D$

GOAL: __________

$97.50 - 104 = 104 - 104 - D$

GOAL: __________

$-6.50 = -D$

GOAL: __________

$$\frac{-6.50}{-1} = \frac{-D}{-1}$$

GOAL: __________

$6.50 = D$

14-16. The following equations have not been solved yet, but GOALS are given for each step. For each equation, copy the GOAL for each step, then do the math to carry out that GOAL. Finally, CHECK to make sure your answer works.

14. TARGET VARIABLE: WAGE $W = 3.5(45) - 12.50$

GOAL: SIMPLIFY

GOAL: SIMPLIFY

CHECK

15. TARGET VARIABLE: HOURS $90.50 = 3.5H - 14.50$

GOAL: ISOLATE

GOAL: SIMPLIFY

GOAL: REDUCE

GOAL: SIMPLIFY

CHECK

16. TARGET VARIABLE: DEDUCTION $102 = 3.5(33) - D$

GOAL: SIMPLIFY

GOAL: ISOLATE

GOAL: SIMPLIFY

GOAL: REDUCE

GOAL: SIMPLIFY

CHECK

17-20. Solve the equation: For each step, give the GOAL, then do the math. Remember to CHECK your answer.

17. $W = 3.5(120) - 17.50$

18. $120 = 3.5H - 34.00$

19. $95 = 3.5H - 13.50$

20. $120 = 3.5(45) - D$

SOLVING EQUATIONS:
PRACTICE USING SIMPLIFY, REDUCE, ISOLATE AND COLLECT

1-10. For each equation, COLLECT the variables. You don't need to solve the whole equation.

Example:

$$2X + 6X = 14$$

$$8X = 14$$

1. $18X - 13X = 150$

2. $75 = 18S + 13S$

3. $25H + 6H - 7H + H = 225$

4. $1.4P - 3.5P + 14 - P = 18.4$

5. $12 = \frac{2}{3} R + \frac{1}{3} R + R$

6. $5T - 6T - T + 4T = 432$

7. $42 = 3C + 5C - 2 + 3C$

8. $6.7 = 8W + 9W - 3W - 12W$

9. $5Y - 7Y - 3Y + 12 + 9Y = 48$

10 $4.25G + 6.5G + .5G - 18G = 105$

11-13. Solve the equation: For each step, give the GOAL, then do the math. Be sure to CHECK your answer.

Example:

$$18 = 2X + 6X - 14$$

GOAL: COLLECT

$$18 = 8X - 14$$

GOAL: ISOLATE

$$18 + 14 = 8X - 14 + 14$$

GOAL: SIMPLIFY

$$32 = 8X$$

GOAL: REDUCE

$$\frac{32}{8} = \frac{8X}{8}$$

GOAL: SIMPLIFY

$$4 = X$$

CHECK:

$$18 = 2(4) + 6(4) - 14$$
$$18 = 8 + 24 - 14$$
$$18 = 18$$

11. $3Y + 9 = 15$

12. $7 + 4S = -1$

13. $11 = 6L - 13$

14-16. Solve the equation. You do NOT need to give the GOAL for each step. CHECK your answer.

14. $-12 = 12R + 24$

15. $5T - 56 + 2T = 7$

16. $6W = 42$

SOLVING PROBLEMS WITH EQUATIONS

NAMES: ______________________ ______________________

DATE: __________ TEACHER: ______________ PERIOD: ____

*** PLEASE DO AS MANY OF THE PROBLEMS AS YOU HAVE TIME FOR ***

1. PAY CHECK, Problem 1

What was the TARGET VARIABLE? __________________________
Explain your answer in a short sentence:

2. PAY CHECK, Problem 2

What was the TARGET VARIABLE? __________________________
Explain your answer in a short sentence:

3. PAY CHECK, Problem 3

What was the TARGET VARIABLE? __________________________
Explain your answer in a short sentence:

4. PIZZA HEAVEN, Problem 1

What was the TARGET VARIABLE? __________________________
Explain your answer in a short sentence:

5. PIZZA HEAVEN, Problem 2

What was the TARGET VARIABLE? __________________________
Explain your answer in a short sentence:

6. PIZZA HEAVEN, Problem 3

What was the TARGET VARIABLE? __________________________
Explain your answer in a short sentence:

7. T-SHIRTS, Problem 1

What was the TARGET VARIABLE? ____________________
Explain your answer in a short sentence:

8. T-SHIRTS, Problem 2

What was the TARGET VARIABLE? ____________________
Explain your answer in a short sentence:

9. T-SHIRTS, Problem 3

What was the TARGET VARIABLE? ____________________
Explain your answer in a short sentence:

SOLVING EQUATIONS: SIMPLIFY, REDUCE, ISOLATE AND COLLECT

Read the paragraph about renting a limousine. Then solve problems 1 and 2.

Your sister and her senior friends rented a limo for last year's prom. To find out how much they would have to spend, they asked about hourly rates at each company, and decided how much to tip the driver and how long to keep the limo. The word equation that describes the situation is:

HOURS * Hourly RATE + TIP = Total COST

Problem 1: The rate for AAA Limo's service was $55 per hour. Your sister and her friends tipped the driver $50, and ended up spending $490 all together. How many hours did they rent the limo?

A. What symbolic equation can you make from the word equation?

B. What is the TARGET VARIABLE in problem 1?

C. Substitute the numbers for hourly rate, tip and total cost, and write the equation you must solve.

D. What is the first GOAL in solving the equation?

E. Carry out the first GOAL and then solve the rest of the equation.

F. How many hours did your sister and her friends rent the limo? (Write a sentence to explain your answer.)

Problem 2: Your sister and her friends decide to use the limo for ten hours at a rate of $55 dollars per hour and tip the driver $50. They want to know how much they will spend.

A. What is the TARGET VARIABLE in problem 2?

B. Substitute the numbers for hours, hourly rate and tip, and write the equation you must solve.

C. What is your first GOAL in solving the equation?

D. Carry out the first GOAL and then solve the rest of the equation.

E. Write a sentence to explain your answer.

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3-9. Copy the equation and write the GOAL that would be the first step in solving the equation. You do not have to solve. Just list the first GOAL: SIMPLIFY, REDUCE, ISOLATE or COLLECT.

3. X + 3 = 25 1ST GOAL: ________________

4. 5.5P + 3P = 250.5 1ST GOAL: ________________

5. 1.65Q - 4.55 = 75.65 1ST GOAL: ________________

6. 456.7 = X + .65 1ST GOAL: ________________

7. 463D = 59,230 1ST GOAL: ________________

8. 50 = 8M + 6M 1ST GOAL: ________________

9. 3.5N = 4.5 + 6.7 1ST GOAL: ________________

10-12. For each equation, give the operation you would need to carry out the GOAL.

Example:

46.5S - 3 = 14.5

GOAL: ISOLATE

OPERATION: + 3

10. 600T = 2,100

GOAL: REDUCE

OPERATION: ________________

11. 2,105D - 1,200 = 43,000

GOAL: ISOLATE

OPERATION: ________________

12. 840 = 900 - D

GOAL: ISOLATE

OPERATION: ________________

13-18. The following equations have already been solved. For each step, list the GOAL that is being done: SIMPLIFY, REDUCE, ISOLATE or COLLECT.

13.

$47 = 9R + 2$

GOAL: ________

$47 - 2 = 9R + 2 - 2$

GOAL: ________

$45 = 9R$

GOAL: ________

$$\frac{45}{9} = \frac{9R}{9}$$

GOAL: ________

$5 = R$

14.

$26 - 8T = -30$

GOAL: ________

$26 - 8T - 26 = -30 - 26$

GOAL: ________

$-8T = -56$

GOAL: ________

$$\frac{-8T}{-8} = \frac{-56}{-8}$$

GOAL: ________

$T = 7$

15.

$72 = 6M$

GOAL: ________

$$\frac{72}{6} = \frac{6M}{6}$$

GOAL: ________

$12 = M$

16.

$10 - H = 50$

GOAL: ________

$10 - H - 10 = 50 - 10$

GOAL: ________

$-H = 40$

GOAL: ________

$$\frac{-H}{-1} = \frac{40}{-1}$$

GOAL: ________

$H = -40$

17.

$16 + 4W = 24$

GOAL: ________

$16 + 4W - 16 = 24 - 16$

GOAL: ________

$4W = 8$

GOAL: ________

$$\frac{4W}{4} = \frac{8}{4}$$

GOAL: ________

$W = 2$

18.

$7 + 3X = 25$

GOAL: ________

$7 + 3X - 7 = 25 - 7$

GOAL: ________

$3X = 18$

GOAL: ________

$$\frac{3X}{3} = \frac{18}{3}$$

GOAL: ________

$X = 6$

19-21. Copy each equation. Solve it using the suggested GOALS and CHECK your answer.

19. $4R = 44$

GOAL: REDUCE

GOAL: SIMPLIFY

CHECK

20. $X + 9 = -5$

GOAL: ISOLATE

GOAL: SIMPLIFY

CHECK

21. $52 + 21P = 10$

GOAL: ISOLATE

GOAL: SIMPLIFY

GOAL: REDUCE

GOAL: SIMPLIFY

CHECK

SOLVING EQUATIONS:
PRACTICE USING SIMPLIFY, REDUCE, ISOLATE AND COLLECT

1-4. For each equation, give the operation needed to carry out the GOAL.

1. 42W = 636

GOAL: REDUCE

OPERATION: ___________

2. 55H + 50 = 625

GOAL: ISOLATE

OPERATION: ___________

3. 350 = 36U

GOAL: REDUCE

OPERATION: ___________

4. 49 = 131D - 46

GOAL: ISOLATE

OPERATION: ___________

5-8. The following equations have already been solved. For each step list the GOAL that is being done: SIMPLIFY, REDUCE, ISOLATE or COLLECT.

5. $4Y - 6 = 22$

GOAL: ________

$4Y - 6 + 6 = 22 + 6$

GOAL: ________

$4Y = 28$

GOAL: ________

$$\frac{4Y}{4} = \frac{28}{4}$$

GOAL: ________

$Y = 7$

6. $-75 = -25 - 5D$

GOAL: ________

$-75 + 25 = -25 - 5D + 25$

GOAL: ________

$-50 = -5D$

GOAL: ________

$$\frac{-50}{-5} = \frac{-5D}{-5}$$

GOAL: ________

$10 = D$

7. $8 = -15A + 11A$

GOAL: ________

$8 = -4A$

GOAL: ________

$$\frac{8}{-4} = \frac{-4A}{-4}$$

GOAL: ________

$-2 = A$

8. $-9N + 5 + 2N = 12$

GOAL: ________

$-7N + 5 = 12$

GOAL: ________

$-7N + 5 - 5 = 12 - 5$

GOAL: ________

$-7N = 7$

GOAL: ________

$$\frac{-7N}{-7} = \frac{7}{-7}$$

GOAL: ________

$N = -1$

9-11. Copy each equation. Then solve using the suggested GOALS and CHECK your answer.

9. $-8 = 2Y - 3Y$

GOAL: COLLECT

GOAL: REDUCE

GOAL: SIMPLIFY

CHECK

10. $9W - 27 + 4W = 12$

GOAL: COLLECT

GOAL: ISOLATE

GOAL: SIMPLIFY

GOAL: REDUCE

GOAL: SIMPLIFY

CHECK

11. $4 = 3K - 3 - 2K$

GOAL: COLLECT

GOAL: ISOLATE

GOAL: SIMPLIFY

CHECK

12-14. Solve the equation: For each step, give the GOAL, then do the math. Be sure to CHECK your answer.

12. $20 + 2M = 64$

13. $150 = -9X - 6X$

14. $8 = 5G - 2G - 27$

SOLVING PROBLEMS WITH EQUATIONS

NAMES: ______________________ ______________________

DATE: __________ TEACHER: ______________ PERIOD: ____

*** PLEASE DO AS MANY OF THE PROBLEMS AS YOU HAVE TIME FOR ***

1. PAY CHECK, Problem 1

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

2. PAY CHECK, Problem 2

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

3. HEAL THE BAY-II, Problem 1

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

4. HEAL THE BAY-II, Problem 2

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

5. HEAL THE BAY-II, Problem 3

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

6. FISHING TRIP, Problem 1

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

7. FISHING TRIP, Problem 2

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

8. FISHING TRIP, Problem 3

What was the TARGET VARIABLE? ______________________
Explain your answer in a short sentence:

SOLVING EQUATIONS:
SIMPLIFY, REDUCE, ISOLATE, COLLECT AND DISTRIBUTE

Read the paragraph on phone calls. Then solve problems 1 and 2.

> On some long distance phone calls, you pay a certain amount of money for the first few minutes you talk. This amount of money is called a "flat fee", and the first few minutes are called your "minimum time". After the first few minutes, you are charged a cheaper amount for each minute you talk. The word equation for this situation is:
>
> Flat FEE + Minute RATE * (Number of MINUTES - Minimum TIME) = COST
>
> The symbolic equation might be: $F + R*(M - T) = C$

Problem 1: The phone company in your area charges a flat fee of 90 cents for the first 2 minutes on this type of call. After the 2-minute minimum time, you must pay 40 cents per minute. What will an 11-minute call cost you?

A. What is the TARGET VARIABLE in problem 1?

B. Substitute the numbers for fee, rate, minutes and minimum time, and write the equation you must solve.

C. What is the first GOAL in solving the equation?

D. Carry out the GOAL and solve the equation.

E. Write a sentence that explains your answer.

Problem 2: The phone company charges your big brother a whopping $48.10 for calling his girlfriend in Alaska. The flat fee is 90 cents, the minute rate is 40 cents per minute, and the minimum time is 2 minutes. How long did your brother talk?

A. What is the TARGET VARIABLE in problem 2?

B. Substitute the numbers for fee, rate, minimum time and cost, and write the equation you must solve. Write cost, fee and rate in cents.

C. What is your first GOAL in solving the equation?

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3-12. Copy the equation and write the GOAL that would be the first step in solving the equation. You do not have to solve. Just list the first GOAL: SIMPLIFY, REDUCE, ISOLATE, COLLECT or DISTRIBUTE.

3. $45 = 3C + 42 + 12C$ 1ST GOAL: __________

4. $6X - 4 = 12$ 1ST GOAL: __________

5. $5(X + 4) = 40$ 1ST GOAL: __________

6. $72.20 = 3.60H - 4.00$ 1ST GOAL: __________

7. $720 - T = 680$ 1ST GOAL: __________

8. $3W + 8 - 5W = 12$ 1ST GOAL: __________

9. $7 - 3(5T - 10) = 67$ 1ST GOAL: __________

10. $P + .75P = 3650$ 1ST GOAL: __________

11. $4(2N + 5) = 36$ 1ST GOAL: __________

12. $32 = 15 - 2(M-3)$ 1ST GOAL: __________

13-17. Solve each equation by using the suggested GOAL for each step. Be sure to CHECK your answers.

13. $28Z - 9Z = 57$

GOAL: COLLECT

GOAL: REDUCE

GOAL: SIMPLIFY

CHECK

15. $12K - 7K + 15 = 23$

GOAL: COLLECT

GOAL: ISOLATE

GOAL: SIMPLIFY

GOAL: REDUCE

GOAL: SIMPLIFY

CHECK

14. $17 = 12R + 5$

GOAL: ISOLATE

GOAL: SIMPLIFY

GOAL: REDUCE

GOAL: SIMPLIFY

GOAL: CHECK

16. $4G - 7.5 = 13.5$

GOAL: ISOLATE

GOAL: SIMPLIFY

GOAL: REDUCE

GOAL: SIMPLIFY

CHECK

17. $33 = 8.21X + 2.79X$

GOAL: COLLECT

GOAL: REDUCE

GOAL: SIMPLIFY

CHECK

18-21. Solve the equation: For each step, give the GOAL, then do the math. Be sure to CHECK your answer.

18. $47 = 2D + 5$

19. $Y - 5 + 7Y = 43$

20. $-8H + 1 - 2H = 81$

21. $9K - 16 = 20$

22-25. Solve the equation. You do NOT need to give the GOAL for each step. CHECK your answer.

22. $6X + 3 = 25$

23. $3.3 = 8.3H - 7.65$

24. $1{,}234 = .22R - 455.6$

25. $3 + 12N - 10N + 6 = 25$

MODELING REAL WORLD SITUATIONS M20-PAGE 1

SOLVING PROBLEMS WITH EQUATIONS

NAMES: ______________________ ______________________

DATE: __________ TEACHER: ______________ PERIOD: ____

*** PLEASE DO AS MANY OF THE PROBLEMS AS YOU HAVE TIME FOR ***

1. PAY CHECK, Problem 1

What was the TARGET VARIABLE? __________________________
Explain your answer in a short sentence:

2. PAY CHECK, Problem 2

What was the TARGET VARIABLE? __________________________
Explain your answer in a short sentence:

3. HEAL THE BAY-II, Problem 1

What was the TARGET VARIABLE? __________________________
Explain your answer in a short sentence:

4. HEAL THE BAY-II, Problem 2

What was the TARGET VARIABLE? __________________________
Explain your answer in a short sentence:

5. T-SHIRTS, Problem 1

What was the TARGET VARIABLE? __________________________
Explain your answer in a short sentence:

6. T-SHIRTS, Problem 2

What was the TARGET VARIABLE? __________________________
Explain your answer in a short sentence:

7. FISHING TRIP, Problem 1

What was the TARGET VARIABLE? __________________________
Explain your answer in a short sentence:

8. FISHING TRIP, Problem 2

What was the TARGET VARIABLE? __________________________
Explain your answer in a short sentence:

SOLVING EQUATIONS:
PRACTICE USING SIMPLIFY, REDUCE, ISOLATE, COLLECT AND DISTRIBUTE

Read the paragraph about buying a new car. Then solve problems 1 and 2.

Your mother likes new cars. Each year she trades in her old car for next year's model. Naturally, the money she can get for her old car is much less than the amount she paid for it the year before. The money she loses is due to "depreciation"; every year, the worth of her car is decreased by the "depreciation rate." The word equation for this situation is:

NEW price - (NEW price * DEPRECIATION rate) = TRADE-in value

(Notice that NEW PRICE shows up in two different places in the equation.)

As a symbolic equation, you could write: N - (N*D) = T

Problem 1: Your mother now has a 1988 Honda Accord which she wants to trade in for next year's model. If she originally paid $12,540 to buy it new, and the depreciation rate is .40 (40%), how much money will she get on the trade-in?

A. What is the TARGET VARIABLE in problem 1?

B. Substitute the numbers for new price and depreciation rate, and write the equation you must solve. Write depreciation rate as a decimal.

C. Solve the equation.

D. Write a sentence explaining your answer.

Problem 2: In 1982, your mother traded in her '81 Civic and bought an '82 Accord. If the depreciation rate was .40, and she got $2,400 for the trade-in, how much did she pay for the Civic when she bought it new in 1981?

A. What is the TARGET VARIABLE in problem 2?

B. Substitute the numbers for depreciation rate and trade-in value, and write the equation you must solve. Write depreciation rate as a decimal.

C. What is your first GOAL in solving the equation?

D. Carry out the first GOAL and finishing solving the problem.

E.Write a sentence explaining your answer.

3-12. For each equation, DISTRIBUTE in order to remove the parentheses. You don't need to solve the whole equation.

Example: $16 + 5(X-3) = 493$

$16 + 5X - 5(3) = 493$

$16 + 5X - 15 = 493$

3. $18 = -6(4W + 1)$

4. $2(3Y - 7) = 56$

5. $-10 = -9(6 + U) - 2U$

6. $-63 = 13H + 7(3H - 1)$

7. $32 = 15 - (4M - 5)$

8. $-5P - 5(6 - 2P) = 0$

9. $-2(4 - K) + 9 = -13$

10. $6(R-2) + 4(R-2) = -36$

11. $5(4 + 2M) - 8(M + 12) = 68$

12. $2(5-3X) + 9 = 28$

13-16. Solve the equation: For each step, give the GOAL, then do the math. Be sure to CHECK your answer.

13. $3(W + 4) = 18$

14. $630 = 40(M - 3) + 230$

15. $-3(2 + A) + 18 = -30$

16. $15(F + 1) - 3F = 759$

17-21. Solve the equation: You do NOT need to list the GOALS. Be sure to CHECK your answer.

17. $2 = 4(D + 1) - 10$

18. $3(2D + 5) - 3 = 42$

19. $20T - 49T + 3 = 975$

20. $40 = 2(4R - 6) + 5R$

21. $4(X - 3.5) + X - .2 = 9.3$

Appendix B

EVALUATION MEASURES: ARITHMETIC TEXT, PRE / POST TEST, STUDENT COURSE EVALUATION, AND UNIT EVALUATIONS

ARITHMETIC TEST

ARITHMETIC SKILLS CHECK-UP

DATE: ____________________

NO NAMES PLEASE!

STUDENT ID: _________________

TEACHER: _____________________

PERIOD: ______________________

Addition

```
_________ 1.    35.08                      _________ 2.    3   3
                 3.21                                      - + -
                 0.0234                                    8   5
              +  9.0
              ---------
```

Subtraction

```
_________ 3.     3621                      _________ 4.            3
               - 1468                                      9  -  5 -
               ------                                              4
```

Multiplication

```
_________ 5.     462.5                     _________ 6.    1     7
               x   7.2                                     -  *  -
               -------                                     3     8
```

Division

```
--------- 7.   .08   0.648                 _________ 8.    2     3
                                                           -     -
                                                           5     5
```

Reduce the fractions:

```
_________ 9.    14                         _________ 10.   225
                --                                         ---
                35                                          15
```

PRE/POST TEST

RAND ALGEBRA TUTOR PROJECT

CONGRATULATIONS! Your class has been selected to participate in a special project. A research team from The RAND Corporation wants you to work with them in developing computer programs for students learning math.

We'd like you to complete this survey today. The survey asks about your background and your opinions concerning math and computers. Some of the questions also ask about algebra. (Algebra involves using equations to solve math problems.) These questions will give us an idea of how much math you know already.

YOUR TEACHER WILL NOT SEE ANY OF YOUR ANSWERS IN THIS SURVEY, AND YOUR ANSWERS WILL NOT AFFECT YOUR COURSE GRADE.

THANK YOU FOR YOUR HELP!

DATE: ________________

STUDENT I.D.: ________

TIME STARTED: ________

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PART 1: BACKGROUND QUESTIONS

DIRECTIONS: Please fill in the parentheses next to each question with the number that matches your answer. Please select only ONE answer to each question.

1. What is your sex? ( )

    (1) male
    (2) female

2. How old are you? __________ years

3. What grade are you in? ( )

    (1) 9th grade
    (2) 10th grade
    (3) 11th grade
    (4) 12th grade

4. What grade do you think you'll get in this course? ( )

    (1) A
    (2) B
    (3) C
    (4) D
    (5) F

DIRECTIONS: We would like to know about your experience with computers. Please circle 1 for "NO" or 2 for "YES" on the following questions.

Have you ever.....	NO	YES
5. taken a course about computers or computer programming?	1	2
6. used computers outside of school, other than for playing video games?	1	2
7. used computers in other classes (for example, in English or math class)?	1	2
8. used a computer to do your homework?	1	2
9. written a computer program?	1	2

PART 2: OPINION QUESTIONS

DIRECTIONS: We want to know what you think about math and computers. Please fill in the parentheses next to each question with the number that matches your answer. There are no right answers, so please tell us what you really think.

10. How do you feel about algebra? ( )

(1) I dislike it
(2) I mildly dislike it
(3) I mildly like it
(4) I like it

11. How difficult is algebra for you? ( )

(1) easy
(2) medium
(3) hard
(4) I have not studied algebra before.

DIRECTIONS: For these questions, circle the number under the answer that best describes what you think or feel. Circle 1 if you "strongly disagree" with the statement, 2 if you disagree, etc.

	Strongly Disagree	Disagree	Agree	Strongly Agree
12. I am good at mathematics.	1	2	3	4
13. Many algebra courses could be improved by the use of computers.	1	2	3	4
14. In math, to know if you have the right answer, you must check with the teacher, a computer, or the book.	1	2	3	4
15. I can get along well in everyday life without using mathematics.	1	2	3	4

DIRECTIONS: Circle the number under the answer that best describes what you think or feel. Circle 1 if you "strongly disagree" with the statement, 2 if you disagree, etc.

	Strongly Disagree	Disagree	Agree	Strongly Agree
16. A problem is easy to figure out even if I've forgotten exactly how to do it.	1	2	3	4
17. In mathematics you can be creative and discover things for yourself.	1	2	3	4
18. There is one right way to solve most algebra problems.	1	2	3	4
19. Using computers to teach algebra is a bad idea.	1	2	3	4
20. Trial and error can often be used to solve an algebra problem.	1	2	3	4
21. It's important to learn algebra because algebra knowledge is useful in everyday life.	1	2	3	4
22. Learning mathematics is mostly memorizing.	1	2	3	4
23. Mathematics is made up of unrelated topics.	1	2	3	4

PART 3: MATH QUESTIONS

DIRECTIONS: The purpose of these questions is to find out how much math you already know. Some of the problems are very difficult, so if you can't solve a problem don't be discouraged, just go on to the next question. Use a calculator if you need one, but please show all your work.

1. Your sister is driving to San Diego with her friends. They know that the distance they can drive on one tank of gas depends on the number of gallons in their gas tank and how many miles their car can get per gallon:

Gallons in tank * Miles per gallon = Distance

If they take your sister's car, which has a nine-gallon tank and gets 27 miles to the gallon, how far can they drive before they run out of gas?

Solve the equations:

2. $4H + 1 = 5$

3. $39W + 21W = 240$

DIRECTIONS: For problems 4 and 5, you *don't* need to do the work. Just decide what the first step in solving the equation should be. Circle the letter that gives the best answer.

4. To solve this equation, what should your first step be?

Equation: $4X + 1 = 2$

First step (circle one):

A. remove the number added to the 4X (isolate)

B. do the arithmetic or reduce fractions (simplify)

C. remove the number multiplied by the X (reduce)

D. remove the parentheses (distribute)

E. collect the X's (collect)

5. To solve this equation, what should your first step be?

Equation: $3(H - 2) = 10$

First step (circle one):

A. remove the number subtracted from the H (isolate)

B. do the arithmetic or reduce fractions (simplify)

C. remove the number multiplied by the H (reduce)

D. remove the parentheses (distribute)

E. collect the H's (collect)

6. Some Santa Monica High School students are putting on a benefit concert for hunger relief in Ethiopia. They want to raise as much money as they can, so they look carefully at all the things their benefit money depends on: the number of people who will come to the concert, the price for each ticket, and the cost of renting a hall. (They don't worry about the cost of the band, because the band will give its time free.)

A. In this situation, which things (variables) are the students thinking about as they organize the concert?

B. Please write an equation based on this situation, which will show how much money the group can raise.

Solve the equations:

7. $3(Y - 2) = 12$

8. $5.2T - 10.3 = 10.5$

9. Another group of Samohi students want to put out a new school sports magazine. They know that the money they can take in for each issue of the magazine is given by this equation:

(Magazines sold * Price of magazine) + Advertising = Money earned

For the first issue, the students get $67.50 in advertising from sport equipment and clothing stores at Santa Monica Place. If they set the price of the magazine at $1.25, and they want to take in $250 altogether, how many magazines must they sell? Use the equation (and a calculator, if you need one) to get your answer.

Solve the equations:

10. $1.3X + 3.2X = 9.0$

11. $.5(P - 2) = 6.05$

TIME FINISHED: _______

THANK YOU VERY MUCH!

STUDENT COURSE EVALUATION

QUESTIONNAIRE ON "MODELLING REAL WORLD SITUATIONS" UNIT

DIRECTIONS: Your answers to these questions will help improve the unit for students in next year's Math A class. Fill in the parentheses next to each question with the number that matches your answer. Please pick only ONE answer to each question.

1. How did you enjoy the unit on modeling real world situations? ( )

   (1) a great deal
   (2) a good bit
   (3) a little
   (4) not at all

2. When math problems were part of situations like "Car Wash" or "BSU-Mecha dance", how interesting did you find them? ( )

   (1) not at all interesting
   (2) somewhat interesting
   (3) interesting
   (4) very interesting

3. How realistic did you think the situations were? ( )

   (1) very realistic
   (2) realistic
   (3) somewhat realistic
   (4) not at all realistic

4. How did you enjoy using the computers? ( )

   (1) a great deal
   (2) a good bit
   (3) a little
   (4) not at all

5. How helpful were the computer's hints when you clicked the mouse on "Help Next Step"? ( )

   (1) Very helpful
   (2) Helpful
   (3) Somewhat helpful
   (4) No help
   (5) Didn't use "Help Next Step"

6. If you could use a computer again for learning things, would you want to use one? ( )

   (1) no
   (2) uncertain
   (3) yes

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7. How easy was it to learn to use the computers? ( )

(1) very difficult
(2) somewhat difficult
(3) fairly easy
(4) very easy

8. What helped you learn to use the computer? (Circle all that helped you.)

(1) instructions in class
(2) computer introduction worksheet
(3) help from my partner
(4) help from the teacher
(5) help from the RAND researcher
(6) trying it out by myself
(7) other: ______________________________

9. What did you think were the worst features of the unit on modeling real world situations? (For example, things you thought were confusing or uninteresting.)

10. What did you think were the best features of the unit?

11. You've spent a lot of time working on situations like the car wash, changing money, going to concerts and so on. Please suggest some ideas for realistic situations that could be given to next year's students.

DIRECTIONS: For these questions, circle the number under the answer that best describes what you think. Circle 1 for "No help," 2 for "Somewhat helpful," and so on.

12. How helpful were each of the following in doing the work for this unit?

	No Help	Somewhat Helpful	Helpful	Very Helpful
Working with a partner	1	2	3	4
Computer assignments	1	2	3	4
Seatwork assignments	1	2	3	4

13. How helpful were each of the following in learning to understand a situation mathematically?

	No Help	Somewhat Helpful	Helpful	Very Helpful
Making a word graph	1	2	3	4
Making a table	1	2	3	4
Using a graph	1	2	3	4
Writing a word equation	1	2	3	4
Writing a symbolic equation	1	2	3	4

14. How helpful were each of the following in learning to solve an equation?

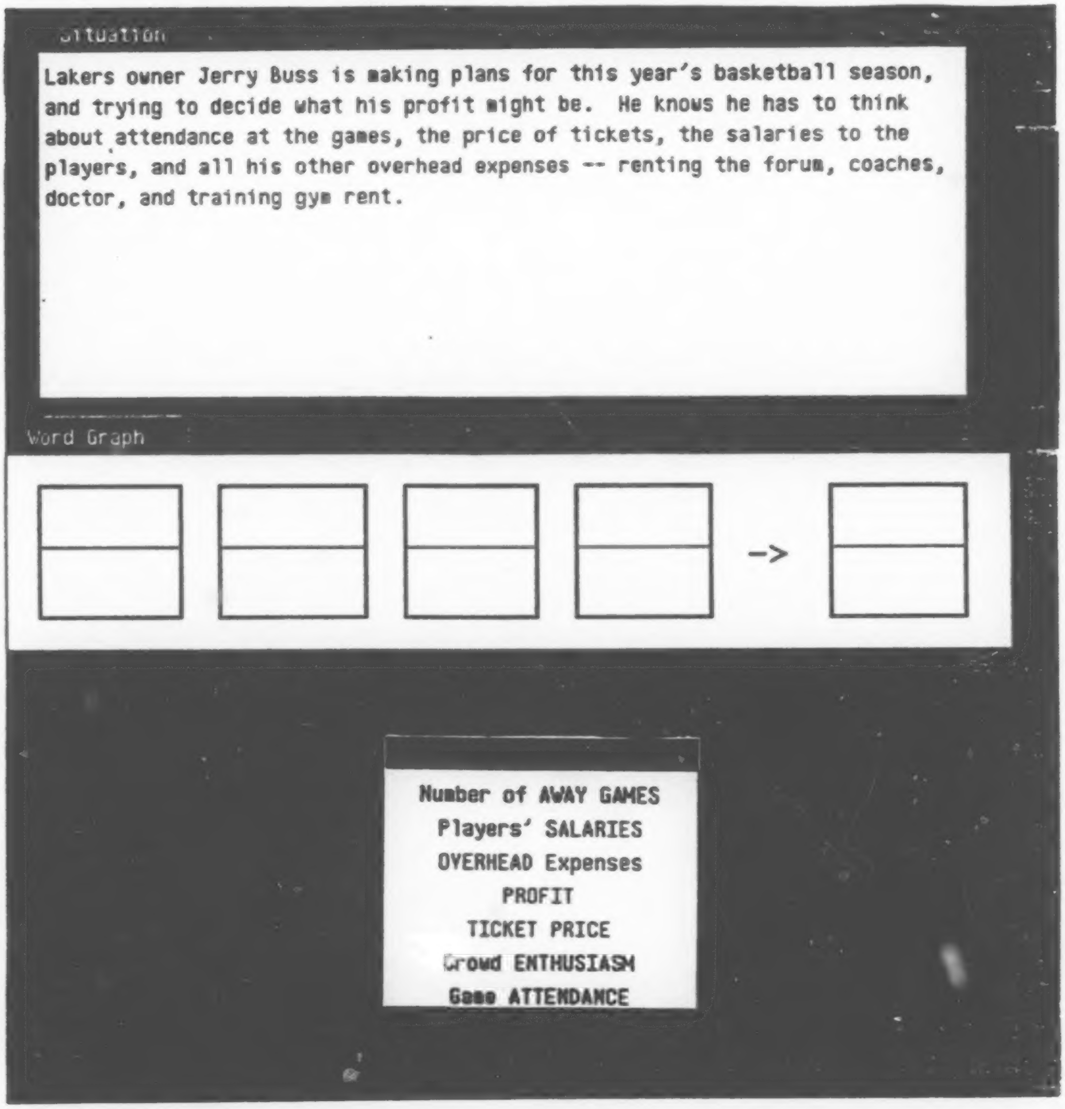
	No Help	Somewhat Helpful	Helpful	Very Helpful
Picking a target variable	1	2	3	4
Naming the goal	1	2	3	4
Naming the operation	1	2	3	4
Doing the math	1	2	3	4

UNIT EVALUATIONS

MODELING REAL WORLD SITUATIONS **EVALUATION 1 - PAGE 1**

Name__________________________ **Period**____________

1. Read the situation. Then choose the variables from the menu and fill in the word graph.



Situation

Lakers owner Jerry Buss is making plans for this year's basketball season, and trying to decide what his profit might be. He knows he has to think about attendance at the games, the price of tickets, the salaries to the players, and all his other overhead expenses -- renting the forum, coaches, doctor, and training gym rent.

Word Graph
->
Number of AWAY GAMES
Players' SALARIES
OVERHEAD Expenses
PROFIT
TICKET PRICE
Crowd ENTHUSIASM
Game ATTENDANCE

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Read the Situation:

Your family is having a garage sale. You have a lot of junk to sell, but you also have to pay for markers and cardboard to make signs. You're lucky because a neighbor has donated a couch to the sale and doesn't want any money for it. Everybody else who helps will get some of the profit.

Word graph:

Junk sold Supplies Couch donation ===> Profit

2. Look at the table:

JUNK SOLD	-	SUPPLIES	+	COUCH DONATION	=	PROFIT
$500		$10		$40		$530
500		10		60		550
500		10		75		560

In this table, only COUCH DONATION changes. What effect does increasing the couch donation have on profit? (Circle one.)

Profit increases Profit decreases No effect

3. Look at the table:

JUNK SOLD	-	SUPPLIES	+	COUCH DONATION	=	PROFIT
500		5		60		555
500		10		60		545
500		15		60		540

In this table, the amount spent on SUPPLIES changes. What effect does increasing the supplies have on profit? (Circle one.)

Profit Increases Profit decreases No effect

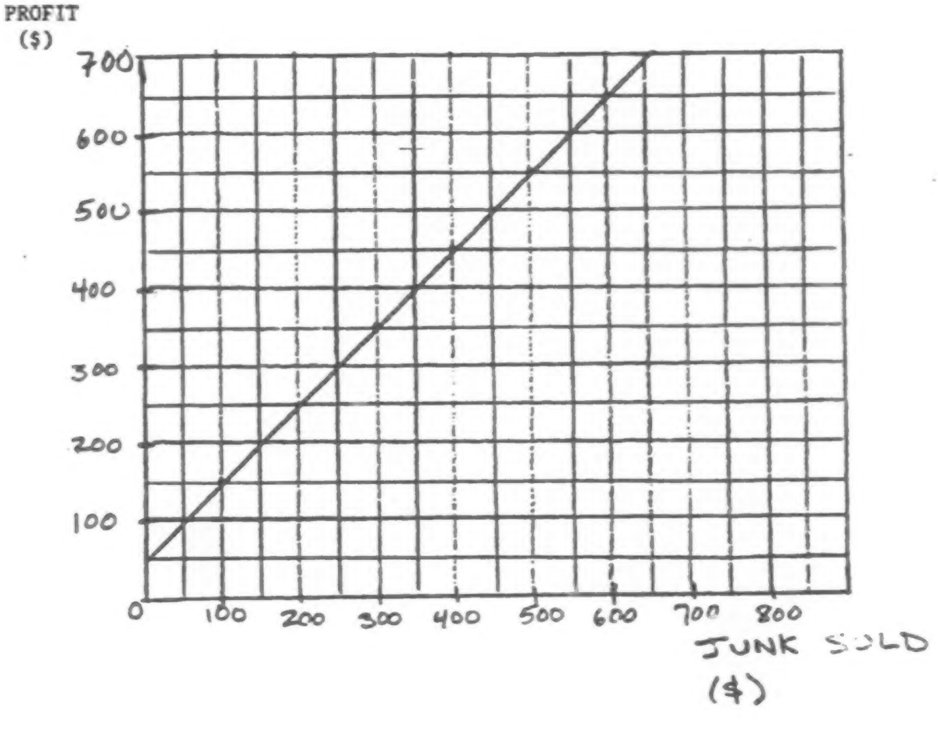
4. Look at the table. How much junk would you have to sell to make a profit of $550.00?

Answer: ____________

Junk sold	-	Supplies	+	Couch price	=	Profit
$300	-	$15	+	$65	=	$350
400	-	15	+	65	=	450
500	-	15	+	65	=	550
600	-	15	+	65	=	650

5. Use the graph to estimate: How much junk do you need to sell to make a profit of $300.00?

Answer: ____________



6. If supplies cost $15, the couch donation is $65, and you wanted to make a profit of $400.00, how much junk would you have to sell? Try different amounts in the table below until your profit is $400. You can use a calculator to figure out the profit.

Answer: ______________

Profit should
come out to $400

	Junk sold	-	Supplies	+	Couch price	=	Profit
Example:	100	-	15	+	65	=	150
		-	15	+	65	=	
		-	15	+	65	=	
		-	15	+	65	=	
		-	15	+	65	=	
		-	15	+	65	=	
		-	15	+	65	=	

DATE: ____________________
STUDENT ID: ____________________

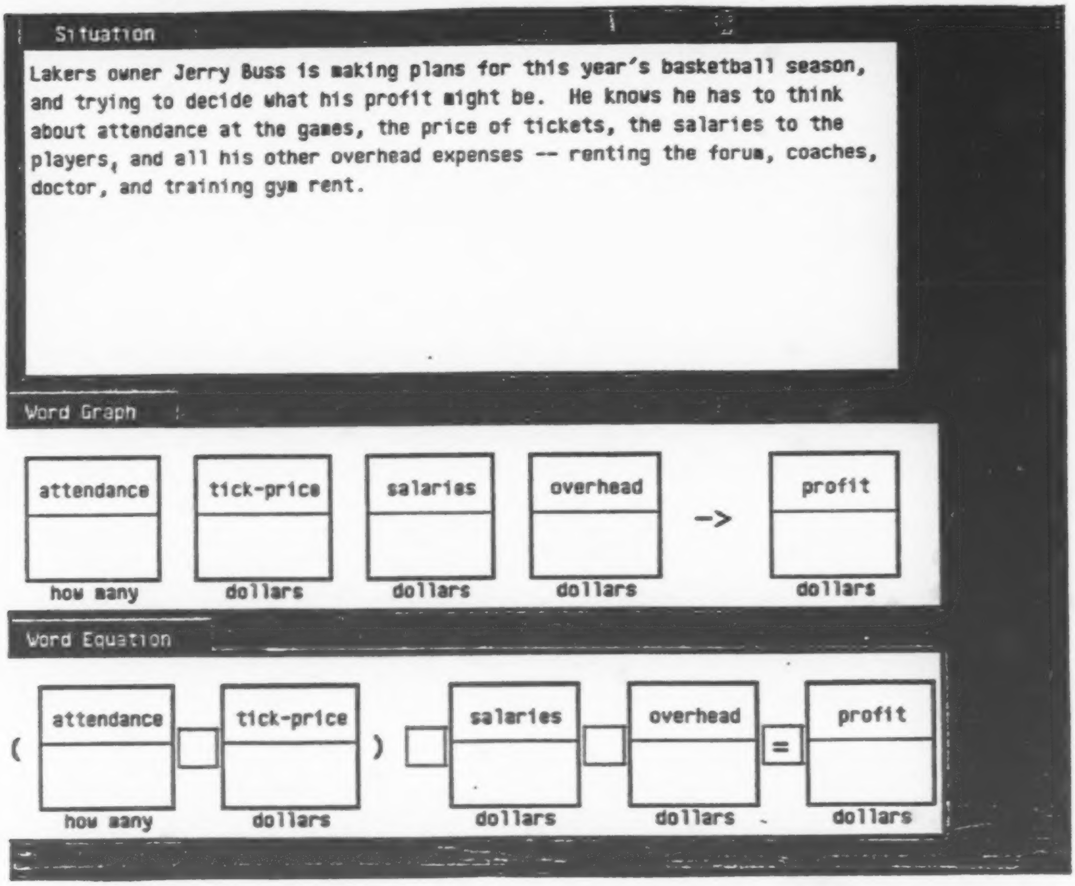
MODELING REAL WORLD SITUATIONS

EVALUATION 2 - PAGE 1

Name____________________

Period__________

1-A. Read the situation. Then fill in the OPERATORS to make a word equation.



Situation

Lakers owner Jerry Buss is making plans for this year's basketball season, and trying to decide what his profit might be. He knows he has to think about attendance at the games, the price of tickets, the salaries to the players, and all his other overhead expenses -- renting the forum, coaches, doctor, and training gym rent.

Word Graph

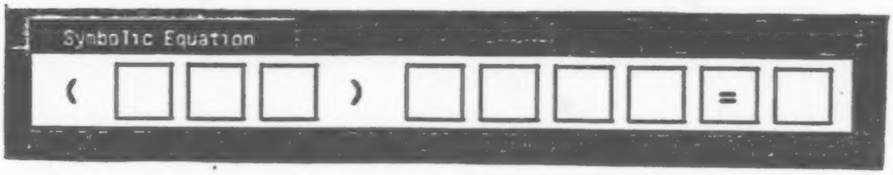
attendance	tick-price	salaries	overhead	->	profit
how many	dollars	dollars	dollars		dollars

Word Equation

(attendance ☐ tick-price) ☐ salaries ☐ overhead = profit

attendance	tick-price	salaries	overhead	profit
how many	dollars	dollars	dollars	dollars

1-B. Make a symbolic equation for the situation with LETTERS and OPERATORS.



Symbolic Equation

(☐ ☐ ☐) ☐ ☐ ☐ ☐ = ☐

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2. Read the problem.

BSU and Mecha are planning a Valentine's Day dance. They want everyone to have a lot of fun, and they also want to raise money for a new popcorn machine for the school. The number of people coming is 200, the ticket price is 3 dollars, and the D.J. costs 75 dollars. What profit will the club make?

Word equation:

(Number of people * Ticket price) - D.J. cost = Profit

Symbolic equation (with given numbers):

$$(200 * 3) - 75 = P$$

A. What is the target variable? ____________

B. What is your goal for the first step?

_________________.

C. Solve the problem.

D. Describe the answer you get in a short sentence.

3. Read the problem.

You're going out for ice cream with a group of friends. 6 people want cones and the bill is 9 dollars. What was the cost of a cone?

Word equation:

(Number of people * Cone cost) = Total bill

Symbolic equation (with given numbers):

(6 * c) = 9

A. What is your goal for the first step?

____

B. What operation do you need to do? ______________
(Be sure to include the NUMBER you would use in the operation.)

C. Solve the problem and check your answer.

D. Describe the answer you get in a short sentence.

4. Read the problem.

On this trip to the ice cream store, 11 people want ice cream, you have a discount coupon for 3 dollars, and the bill is $19. How much did a cone cost at this store?

Word equation:

(Number of people * Cone cost) - Discount = Total bill

Symbolic equation (with given numbers):

$$11c - 3 = 19$$

A. What is your goal for the first step?

____

B. What operation do you need to do? ______________
(Be sure to include the NUMBER you would use in the operation.)

C. Solve the problem and check your answer.

D. Describe the answer you get in a short sentence.

5. Read the problem.

> You're buying food for a pre-dance party. Drinks are 2 dollars a person, and sandwiches are 4 dollars a person. If the total cost of the party must be no more than 90 dollars, how many people can you feed at the party?
>
> Word equation:
>
> (Drink cost * People) + (Sandwich cost * People) = Total cost

Symbolic equation (with given numbers):

$$2p + 4p = 90$$

A. What is the target variable? ____________

B. What is your goal for the first step?

____

C. Solve the problem and check your answer.

D. Describe the answer you get in a short sentence.

6. Read the problem.

Your family has won $2,500 to take a dream vacation to Hawaii. There are 4 people in the family, and airplane tickets cost 365 dollars for each person. How much can be spent on each person for other expenses like hotel and food?

Word equation:

People in family * (Ticket cost + Expenses) = Total trip cost

Symbolic equation (with given numbers):

$$4\,(365 + e) = 2500$$

A. What is your goal for the first step?

____

B. Solve the problem and check your answer.

C. Describe the answer you get in a short sentence.

Solve the equation and check your answer. You do NOT need to give goals for each step.

7. $3Y + 9 = 21$

8. $36 - 11T = -8$

9. $124 = -9X + 40X$

10. $5(P - 2) + 45 = 60$

BIBLIOGRAPHY

Brown, J. S., A. Collins, and P. Duguid (1988). *Cognitive Apprenticeship, Situated Cognition, and Social Interaction*, Institute for Research on Learning, IRL888-0008, Bolt Beranek and Newman, Cambridge, Massachusetts.

California State Department of Education (1985). *California Mathematics Framework*, Sacramento, California.

Campbell, D. T., and J. C. Stanley (1966). *Experimental and Quasi-Experimental Designs for Research,* Rand McNally, Chicago, Illinois

Carnegie Forum on Education and the Economy, Task Force on Teaching as a Profession (1986). *A Nation Prepared,* New York.

Carpenter, T. P., M. K. Corbitt, H. S. Kepner, M. M. Lindquist, and R. E. Reys (1980). "Students' Affective Responses to Mathematics: Secondary School Results from National Assessment," *Mathematics Teacher*, October, pp. 531–539.

Cook, T. D., and D. T. Campbell (1979). *Quasi-Experimentation: Design and Analysis Issues for Field Settings,* Rand McNally Publishing, Chicago, Illinois.

Clement, J. (1982). "Algebra Word Problem Solutions: Thought Processes Underlying a Common Misconception," *Journal of Research in Mathematics Education,* Vol. 13, pp. 16–30.

Collins, A., J. S. Brown, and S. E. Newman (1989). "Cognitive Apprenticeship: Teaching the Craft of Reading, Writing, and Mathematics," in L. B. Resnick (ed.), *Cognition and Instruction: Issues and Agendas,* Lawrence Erlbaum Associates, Hillsdale, New Jersey.

Dossey, J. A., I. Mullis, M. M. Lindquist, and D. L. Chambers (1988). *The Mathematics Report Card, Are We Measuring Up? Trends and Achievement Based on the 1986 National Assessment,* Educational Testing Service, Report N. 17-M-01, Princeton, New Jersey.

Fey, J. T., et al. (eds.) (1984). *Computing and Mathematics: The Impact on Secondary School Curricula,* Report of the National Council of Teachers of Mathematics Conference, National Council of Teachers of Mathematics, Inc., Reston, Virginia.

Fey, J. T. (1986). *Algebra with Computers*, University of Maryland, College Park, Maryland.

Flagg, B. N. (1990). *Formative Evaluation for Educational Technologies,* Lawrence Erlbaum Associates, Hillsdale, New Jersey.

Kaput, J. J. (1989). "Linking Representations in the Symbol of Algebra," in S. Wagner and C. Kieran (eds.), *Research Issues in the Learning and Teaching of Algebra*, Lawrence Erlbaum Associates, Hillsdale, New Jersey.

Leinhardt, G. (1989). "Graphing and Functions: Cases of Learning to Teach, and Teaching," paper presented at the annual meeting of the American Educational Research Association, San Francisco, California.

McArthur, D. (1987). "Developing Computer Tools to Support Learning and Performing Complex Cognitive Tasks," in D. Berger and C. Pedzek (eds.), *Applications of Cognitive Psychology: Computing and Education*, Lawrence Erlbaum Associates, New Jersey. (Also published as RAND, N-2980-NSF.)

McArthur, D., C. Stasz, and J. Hotta (1987). *Learning Problem-Solving Skills in Algebra*. RAND, N-2595-NSF.

McArthur, D., C. Stasz, J. Hotta, O. Peter, and C. Burdorf (1988). "Skill-Oriented Task Sequencing in an Intelligent Tutor for Basic Algebra," *Instructional Science*, Vol. 17, pp. 281–307. (Also published as RAND, N-2966-NSF.)

McArthur, D., and C. Stasz (1990). *An Intelligent Tutor for Basic Algebra*, RAND, R-3811-NSF.

National Commission on Excellence in Education (1983). *A Nation at Risk*, Washington, D.C., U.S. Government Printing Office.

Newman, D. (1989). "Formative Experiments on Technologies That Change the Organization of Instruction," paper presented at the annual meeting of the American Educational Research Association, Chicago, Illinois.

Office of Technology Assessment (1988). *Power On! New Tools for Teaching and Learning*, Washington, D.C., U.S. Government Printing Office.

Resnick L. (1987). *Education and Learning to Think*, National Academy Press, Washington, D.C.

Robyn, A., C. Stasz, T. Ormseth, and D. McArthur (1989). "Implementing Computer-Assisted Instruction in First-Year Algebra Classes," paper presented at the annual meeting of the American Educational Research Association, San Francisco, California.

Romberg, T. (1987). *Curriculum and Evaluation Standards for School Mathematics, Working Draft*, National Council of Teachers of Mathematics, Reston, Virginia.

Salomen, G. (1985). "Information Technologies: What You See Is Not (Always) What You Get," *Educational Psychologist*, Vol. 20, No. 4, pp. 207–216.

Schoenfeld, A. H. (1985). "Students' Beliefs About Mathematics and Their Effects on Mathematical Performance: A Questionnaire Analysis," paper presented at the annual meeting of the American Educational Research Association, Chicago, Illinois.

Schofield, J. W., and D. Verban (1988). "Computer Usage in the Teaching of Mathematics: Issues Which Need Answers," in D. Grouws and T. Cooney (eds.), *Perspectives on Research on Effective Mathematics Teaching*, Vol. 1, National Council of Teachers of Mathematics, Inc., Reston, Virginia.

Shavelson, R. J., C. Stasz, S. Schlossman, N. Webb, J. Y. Hotta, and S. Goldstein (1986). *Evaluating Student Outcomes from Telecourse Instruction: A Feasibility Study,* RAND, R-3422-CPB.

Slavin, R. E. (1986). *Using Student Team Learning,* Center for Research on Elementary and Middle Schools, The Johns Hopkins University, Baltimore, Maryland.

Stanley, R. (1989). *Math A, A Brief Description,* California State Department of Education, Sacramento, California.

Stasz, C., T. Ormseth, D. McArthur, and A. Robyn (1989). "An Intelligent Tutor for Basic Algebra: Perspectives on Evaluation," paper presented at the annual meeting of the American Educational Research Association, San Francisco, California.

Strudler, N., and M. Gall, (1988). "Successful Change Agent Strategies for Overcoming Impediments to Microcomputer Implementation in the Classroom," paper presented at the annual meeting of the American Educational Research Association, New Orleans, Louisiana.

Thompson, C. S., and E. C. Rathmell, (1988). "NCTM's standard for school mathematics, K-12," *Mathematics Teacher,* Vol. 81, No. 5, pp. 348–351.

Tobin, K. (1987). "Forces Which Shape the Implemented Curriculum in High School Science and Mathematics," *Teaching and Teacher Education*, Vol. 3, No. 4.

Usiskin, Z. (1985). "We Need Another Revolution in Secondary School Mathematics," in *The Secondary School Mathematics Curriculum,* National Council of Teachers of Mathematics.

RAND/N-3326-NSF/RC